Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sportontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 18 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The time (in seconds) between the consecutive landings of aeroplanes at an airport on day 1 was recorded. The data is given in the Cumulative Frequency table below.

Time in seconds	**Number of aeroplanes (Frequency)**	**Cumulative Frequency**
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	k	p
$210 < t \leq 240$	7	80

1.1 Show that $k = 10$. (1)

1.2 Write down the value of p. (1)

1.3 Calculate the estimated mean time between the landings of two consecutive aeroplanes. (3)

1.4 It is given that (q ; 186,89) is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.

1.4.1 Write down the estimated standard deviation of the time between the consecutive landings of the aeroplanes. (2)

1.4.2 Calculate the value of q. (1)

1.5 On day 2, the same number of aeroplanes that landed on day 1, land at the airport. The elapsed time between all the consecutive landings of all the aeroplanes is m seconds shorter than the time that is given in the table above.

If an ogive is to be drawn of the data of day 2, the following will be true:

- The ogive will be grounded at (57 ; 0)
- The maximum value of the ogive will be at (237 ; 80)

Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same. (2)

[10]

QUESTION 2

The marks, in percentage, obtained in an Accounting and Mathematics test by a group of ten Grade 12 learners is shown in the table below.

Accounting Test	76	65	88	68	70	79	51	66	59	74
Mathematics Test	80	69	93	19	76	85	57	79	62	78

2.1 Identify an outlier of the above data. (1)

2.2 Determine the equation for the least squares regression line after ignoring the outlier in the above data. (3)

2.3 Another learner in the same class obtained 83% in the Accounting test, but due to illness could not write the Mathematics test. Use the equation established in 2.2 to predict the learnerȏs mark for the Mathematics test. (2)

2.4 The teacher decided to award the learner who was absent the predicted mark obtained in 2.3 for the Mathematics test. Other learners in the class felt that it was unfair.

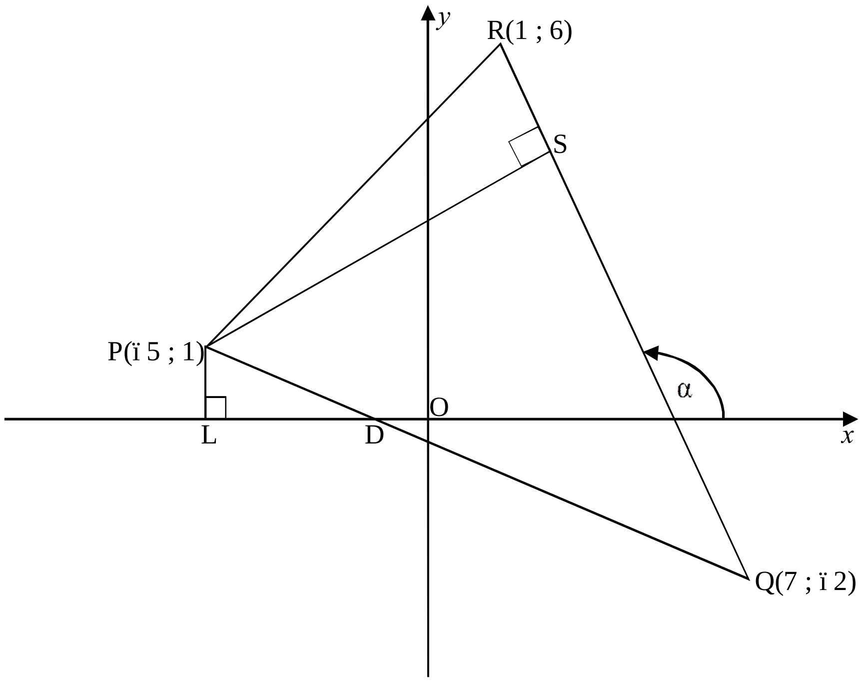
Motivate to these learners why the predicted mark is a good indication of what the learner may have scored in the Mathematics test. (2)

2.5 After the Mathematics subject advisor has moderated the answer books of the Mathematics tests, she decides to lower every test mark by p%. Explain, **without any calculations,** what influence the lowering in the marks of the Mathematics test has on the slope of the least squares regression line of the above data when the outlier is ignored. (2)

[10]

QUESTION 3

In the diagram below, P (ï 5 ; 1), Q(7 ; ï 2) and R(1 ; 6) are the vertices of ΔPQR. PQ intersects the x-axis at D. The angle of elevation of QR is α. PS $\perp$ RQ and L lies on the x-axis such that PL $\perp$ $x -$ axis.

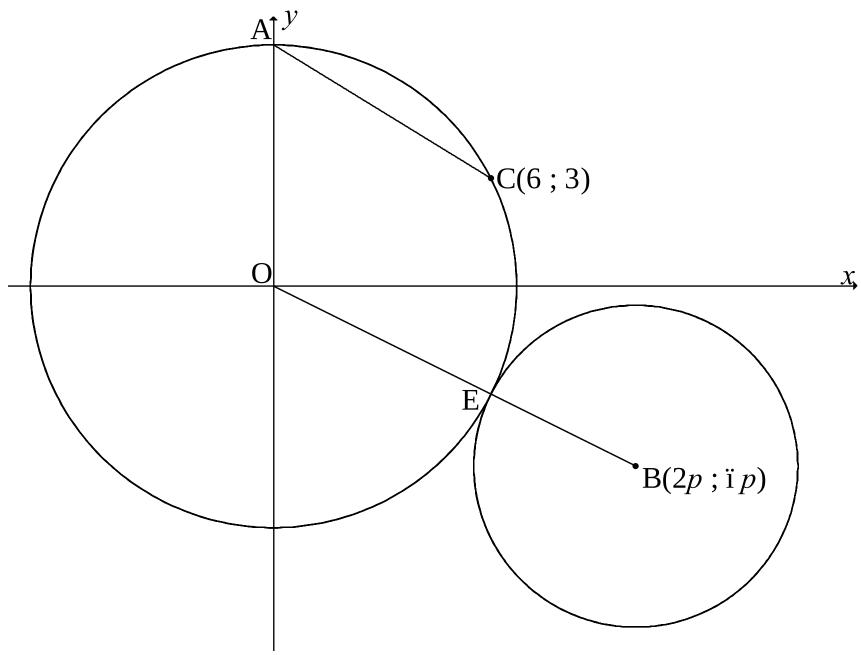


3.1	Write down the equation of the line PL.	(1)
3.2	Calculate the gradient of QR.	(2)
3.3	Determine the equation of the line PS.	(4)
3.4	Calculate the size of the angle of inclination of PQ.	(3)
3.5	Calculate the size of $P\hat{Q}S$.	(4)
3.6	It is given that the areas of ΔPRS $= 4x^2$ and ΔPQS $= 16x^2$. Calculate the length of SQ, WITHOUT calculating the coordinates of S.	(5)

[19]

QUESTION 4

In the diagram below, two circles are given. Circle O, having the origin as centre, intersects the y-axis at A and passes through the point C(6 ; 3). The circle having centre B($2p$; ï p) touches circle O externally in point E. The centres of the two circles are joined by the line OB.



4.1 Determine the equation of the circle having centre O. (2)

4.2 Determine the coordinates of A. (2)

4.3 Determine the equation of AC. (3)

4.4 Calculate the value(s) of k for which the line $y = \frac{1-\sqrt{5}}{2}x + k$ will intersect the circle having centre O at two points, one of which has a positive x-value and the other a negative x-value. (2)

4.5 It is given that the length of $EB = \sqrt{20}$.

4.5.1 Write down, in terms of p, the equation of circle B in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.5.2 Determine the value of p if $p > 0$. (5)

4.6 Suppose a third circle with the following equation is given:
$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$

Determine the maximum length that the radius of this circle can be for any value of θ. (6)

[22]

QUESTION 5

5.1 Simplify each of the following **without the use of a calculator.**
Show ALL calculations.

5.1.1 $\dfrac{\sin 110^\circ . \tan 60^\circ}{\cos 540^\circ . \tan 250^\circ . \sin 380^\circ}$ (7)

5.1.2 $(1-\sqrt{2}\sin 22{,}5^\circ)(\sqrt{2}\sin 22{,}5^\circ + 1)$ (4)

5.2 Given the expression: $\dfrac{\cos 2x . \tan x}{\sin^2 x}$

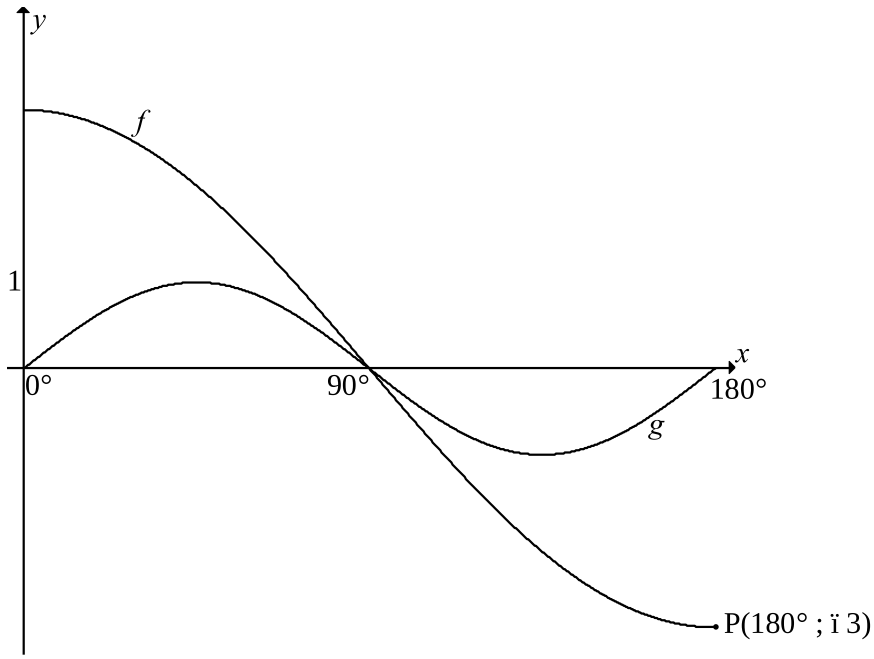
5.2.1 For which value(s) of x, in the interval $x \in [0^\circ\,;180^\circ]$, will this expression be undefined? (3)

5.2.2 Prove that $\dfrac{\cos 2x . \tan x}{\sin^2 x} = \dfrac{\cos x}{\sin x} - \tan x$ (5)

[19]

QUESTION 6

In the diagram below, the graphs of $f(x) = a\cos x$ and $g(x) = \sin bx$ are drawn for the interval $x \in [0° ; 180°]$. The point P(180° ; ï 3) is on the graph of *f*.



6.1 Write down the values of a and b. (2)

6.2 Write down the period of *f*. (1)

6.3 Write down the range of $g(x) + 3$ (2)

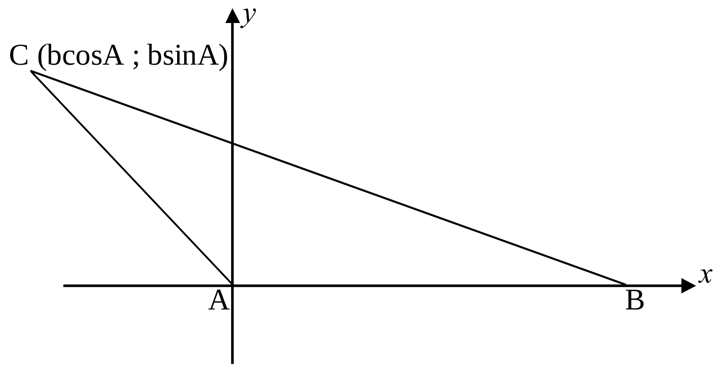
6.4 For which values of x, in the given interval, is $f(x).g'(x) > 0$ (3)

6.5 When the graph of g is shifted $q°$ to the left, it coincides with the function $y - \cos^2 x = -\sin^2 x$. Determine the value of q. (3)

[11]

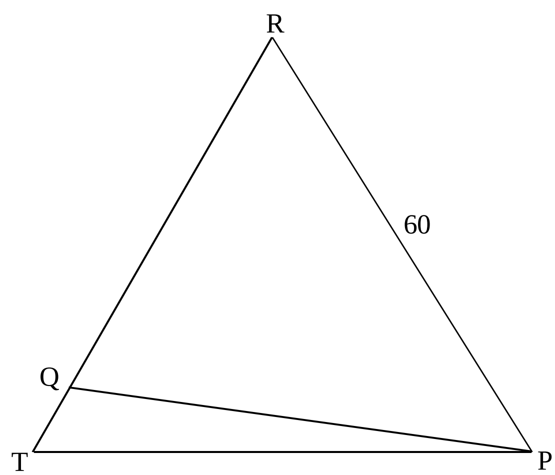
QUESTION 7

7.1 In the diagram below, ΔABC is drawn having A at the origin, B on the x-axis and the vertex C has the coordinates (bcosA ; bsinA).



Use the above diagram to prove that $a^2 = b^2 + c^2 - 2bc\cos A$ (4)

7.2 In the diagram below, Δ TPR is equilateral with PR = 60 units. Q is a point on RT such that RQ:QT = 5:1.



7.2.1 Show, by calculations, that PQ = 55,68 units. (4)

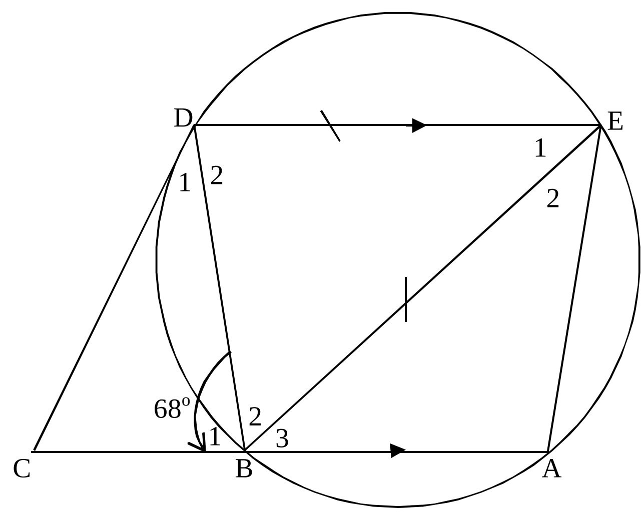
7.2.2 It is given that S is any point on the straight line PQ. Calculate the distance QS when S is the nearest to R. (4)

[12]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

In the diagram below, BAED is a cyclic quadrilateral with BA || DE. BE = DE and $D\hat{B}C = 68^\circ$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

8.1.1 $D\hat{E}A$ (2)

8.1.2 $\hat{A}$ (1)

8.1.3 $\hat{D}_2$ (2)

8.1.4 $\hat{B}_2$ (1)

8.1.5 $\hat{D}_1$ (3)

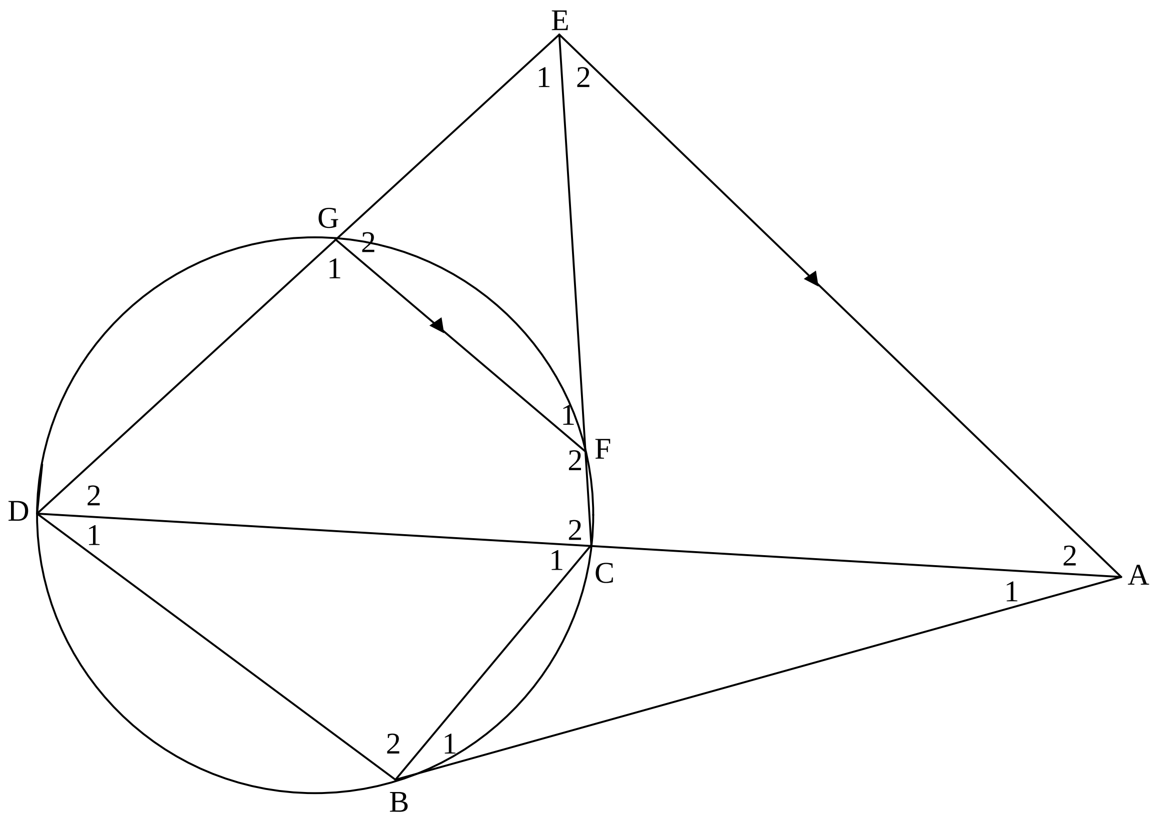
8.2 Prove that ΔBDC is isosceles. (2)

8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)

[13]

QUESTION 9

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. EA | | GF



9.1 Give a reason why $\hat{B}_1 = \hat{D}_1$. (1)

9.2 Prove $\Delta ABC \,|||\, \Delta ADB$. (3)

9.3 Prove $\hat{E}_2 = \hat{D}_2$ (4)

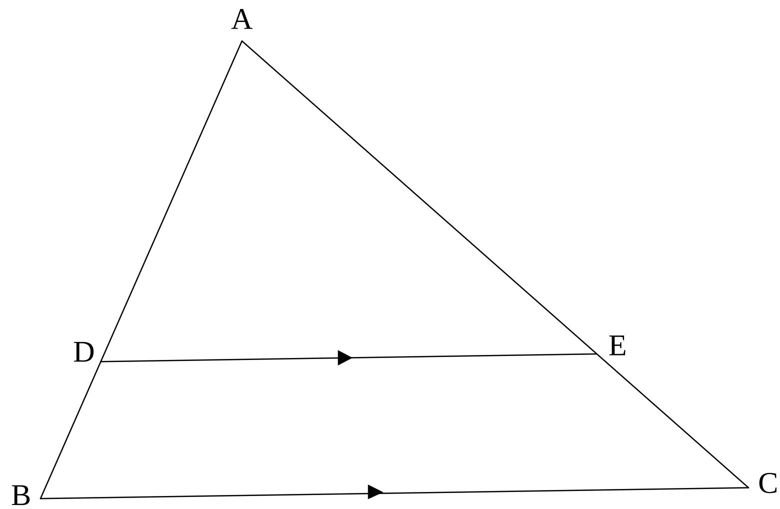
9.4 Prove $AE = \sqrt{AD \times AC}$. (5)

9.5 Hence, show that AE = AB. (3)

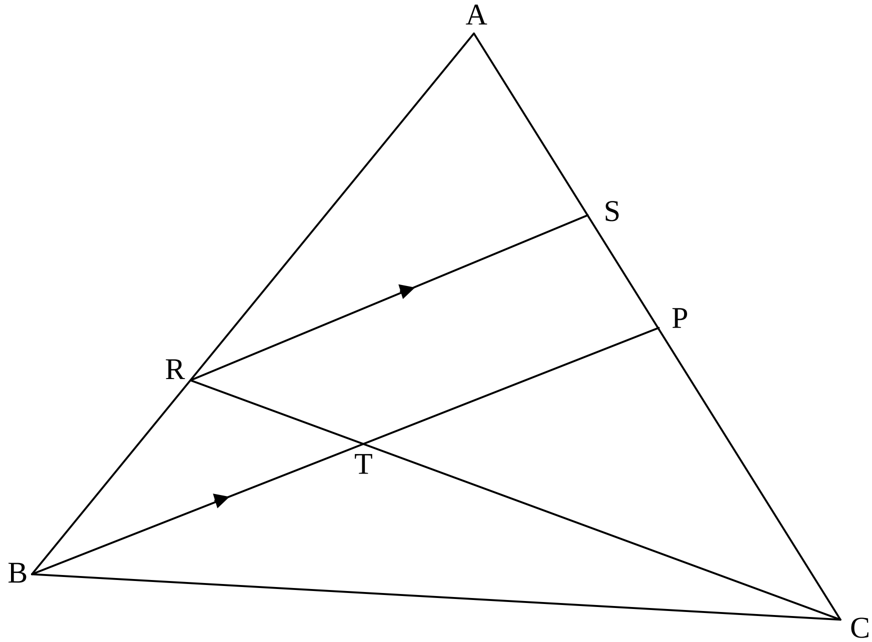
[16]

QUESTION 10

10.1 In ΔABC below, D and E are points on AB and AC respectively such that DE | | BC. Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



10.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS | | BP and $\frac{AR}{AB} = \frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$ (4)

10.2.2 $\frac{RT}{TC}$ (3)

10.2.3 $\frac{\text{Area of } \varphi RAS}{\text{Area of } \varphi RSC}$ (2)

10.2.4 $\frac{\text{Area of } \varphi TPC}{\text{Area of } \varphi RSC}$ (3)

[18]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $\quad S_\infty=\frac{a}{1-r}$; $-1<r<1$;

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $\quad P=\frac{x[1-(1+i)^{-n}]}{i}$ $\quad f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad a^2=b^2+c^2-2bc.\cos A$

$area\,\Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) ï P(A and B)

$\hat{y}=a+bx$

$$b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$