Education and Sport Development

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 18x = 0$ (3)

1.1.2 $7x^2 - 4x = 5$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $(x+5)(x-2) > 0$ (2)

1.1.4 $26 - 5^{2x} = (5^x - 6)^2$ (6)

1.2 Solve simultaneously for x and y:

$x - 4y = 5$ and $3x^2 - 5xy + 2y^2 = 25$ (6)

1.3 Solve for x if: $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$ (4)

[25]

QUESTION 2

2.1 Given the following arithmetic sequence: $-11 ; -4 ; 3 ; \ldots$

Determine the:

2.1.1 General term in the form $T_n = bn + c$. (2)

2.1.2 Value of the 60^{th} term. (2)

2.1.3 Sum of the first 60 terms. (2)

2.2 Hence, or otherwise, write $-4 + 3 + 10 + \ldots + 486$ in sigma notation. (4)

2.3 This arithmetic sequence $-11 ; -4 ; 3 ; \ldots$ forms the first three first differences of a quadratic sequence. Which term in this quadratic sequence will be the smallest? Show all your calculations. (5)

[15]

QUESTION 3

Consider the geometric series: $5(3x+1) + 5(3x+1)^2 + 5(3x+1)^3 + \ldots$

3.1 For which value(s) of x will the series converge? (3)

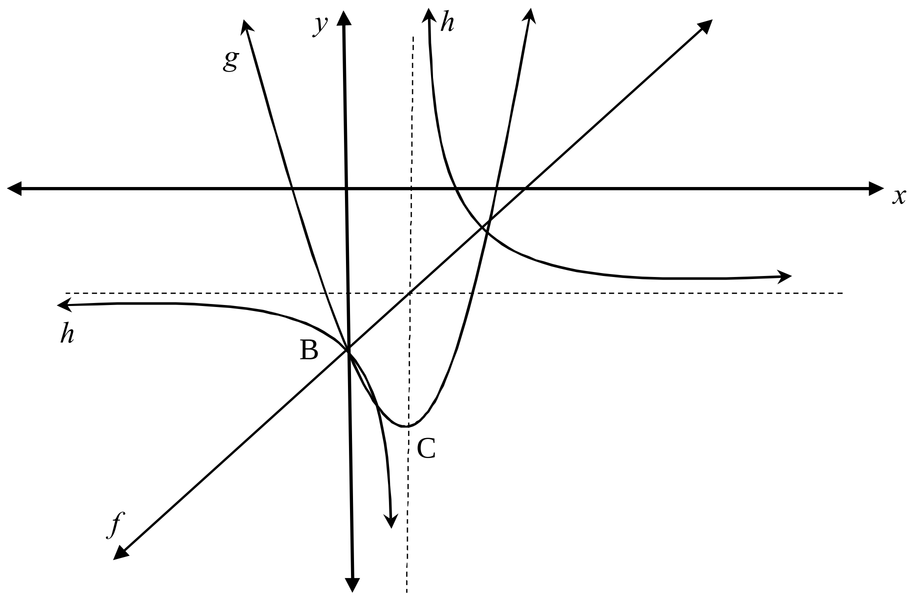
3.2 Calculate the sum to infinity of the series if $x = -\frac{1}{6}$ (4)

[7]

QUESTION 4

The graphs of $g(x)=\frac{1}{2}(x-2)^2-9$ and $h(x)=\frac{a}{x+p}+q$ are sketched below.

The axis of symmetry of graph g is the vertical asymptote of graph h. The line f is an axis of symmetry of graph h. B is the y-intercept of h, g and f.



4.1 Write down the coordinates of C, the turning point of g. (2)

4.2 Determine the coordinates of B. (2)

4.3 Write down the equation of f. (2)

4.4 Determine the equation of h. (5)

4.5 Write down the equations of the vertical and horizontal asymptotes of $k(x)=3h(x)-2$. (2)

4.6 Determine the x-intercept of h. (3)

4.7 For which values of x will:

4.7.1 $\frac{g'(x)}{h(x)}\geq 0$ (3)

4.7.2 $f^{-1}(x-1)<2$ (4)

4.8 Calculate the value(s) of k for which $g(x)=f(x)+k$ has two unequal positive roots. (6)

[29]

QUESTION 5

5.1 Consider the function $f(x) = \left(\frac{5}{6}\right)^x$

5.1.1 Write down the equation of h, the reflection of f in the y-axis. (1)

5.1.2 Write down the equation of $f^{-1}(x)$ in the form $y = ...$ (2)

5.1.3 For which value(s) of x will $f^{-1}(x) \geq 0$? (2)

5.2 The function defined as $f(x) = ax^2 + bx + c$ has the following properties:

- $f'(-2,5) = 0$
- $f(1) = 0$
- $b^2 - 4ac > 0$
- $f(-2,5) = 6$

Draw a neat sketch graph of f. Clearly show all x-intercepts and turning point. (4)

[9]

QUESTION 6

On 1 July 2010, David bought a tractor for R2 000 000. On that day, he paid a deposit of 25% of the purchase price and the bank granted him a loan at an interest rate of 9,5% per annum, compounded quarterly, to pay off the balance of the purchase price. David agreed to pay quarterly instalments of R58 000, starting on 1 January 2011.

6.1 How much money did David borrow from the bank? (2)

6.2 How many quarterly instalments are required to pay off the loan? (6)

6.3 Calculate the amount owing on the loan immediately after David paid his quarterly instalment on 1 July 2016, i.e. six years after he bought the tractor. (4)

6.4 Hence, calculate the amount of interest that David had paid on this loan until immediately after paying his quarterly instalment on 1 July 2016. (4)

[16]

QUESTION 7

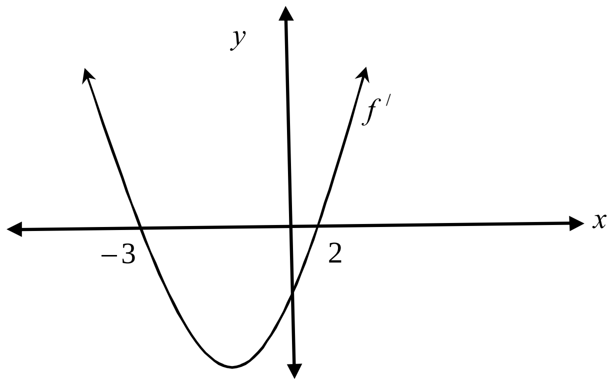
7.1 Given: $f(x) = -x^2 + 3x - 7$
Determine $f'(x)$ from first principles. (6)

7.2 Determine: $D_x\left[15\sqrt[5]{x^4} - \frac{3x^7 + x}{4x^3}\right]$ (6)

[12]

QUESTION 8

The graph of $f^{/}(x) = x^2 + bx + c$, where f is a cubic function, is sketched below.
The derivative function $f^{/}$ cuts the x-axis at $x = -3$ and $x = 2$.



8.1 For which values of x is graph f increasing? (2)

8.2 At which value of x does graph f have a local maximum value? (1)

8.3 Determine the equation of $f^{/}(x)$. (2)

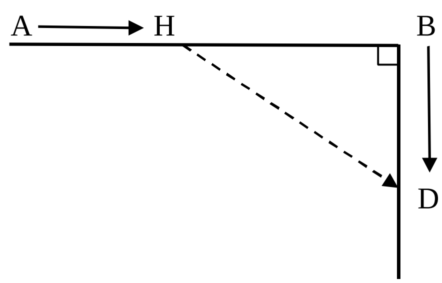
8.4 If $f(x) = px^3 + qx^2 + rx + 10$, show that $p = \frac{1}{3}$, $q = \frac{1}{2}$ and $r = -6$. (4)

8.5 For which value(s) of x is graph f concave down? (3)

[12]

QUESTION 9

A hunter was standing at point A, along the fence of a rectangular game enclosure, when he spotted a deer standing at point B, the corner of the rectangular enclosure. The distance from A to B is 1 200 m. At exactly the same time as the hunter started to move in an easterly direction towards B, the deer started to move in a southerly direction towards D. The hunter moves at 4 metres per second and the deer moves at 5 metres per second. After *t* seconds, the hunter is at a point H and the deer is at point D.



The hunter tries to shoot the deer but with his caliber rifle he must be at most 800 m from the deer.

9.1 Show that the distance between the hunter and the deer (HD) at *t* seconds after they both started moving can be written as:
$\text{HD(t)} = \sqrt{41t^2 - 9\ 600t + 1\ 440\ 000}$ (4)

9.2 How long after they started walking, were they the nearest to one another? Show all calculations. (3)

9.3 The calibre of the hunterøs rifle allows him to be at most 800 m from his target. Was the hunter within shooting range of the deer at the time when they were nearest to each other? Show all calculations. (3)
[10]

QUESTION 10

The rules for the final game of the North West Hockey tournament specify that there must be a winner. In the event of a draw, the winner will be determined by a penalty-flick shootout.

On the day that the final game of the North West Hockey tournament takes place, there is a 45% chance that it could rain, a 32% chance that it could be cloudy or it could be sunny. The team from Taung, a low rainfall area, has an 18% chance of winning the tournament on a rainy day, a 39% chance of winning on a cloudy day and a 63% chance of winning on a sunny day.

10.1 Draw a tree diagram to represent all outcomes of the above information. (2)

10.2 What is the probability of the Taung hockey team winning the final game of the tournament? (4)
[6]

QUESTION 11

A horse breeder has 9 single horse stables in a row next to each other. He has 4 stallions (male horses) and 5 mares (female horses), where one of the stallions is his breeding stallion and one of the mares his breeding mare. The horses are placed randomly in the stables.

11.1 In how many different ways can the 9 horses be placed in the 9 stables? (1)

11.2 In how many different ways can the 9 horses be placed if the breeder wants to place the breeding stallion and breeding mare next to each another? (2)

11.3 What is the probability that there will be a mare placed on both ends of the row stables? (3)

11.4 If 5 stables became unavailable due to renovations, in how many different ways can the breeder place his horses in the remaining single stables such that there will be at least one mare in these stables? (3)

[9]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$