education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

PROVINCIAL ASSESSMENT/
PROVINSIALE ASSESSERING

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

JUNE/*JUNIE 2024*

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS: 150/*PUNTE: 150*

These marking guidelines consist of 16 pages.
Hierdie nasienriglyne bestaan uit 16 bladsye.

NOTE

- **If a candidate answers a question twice, only mark the FIRST attempt/** ***Indien `n kandidaat `n vraag twee keer antwoord, word slegs die EERSTE poging gemerk./***

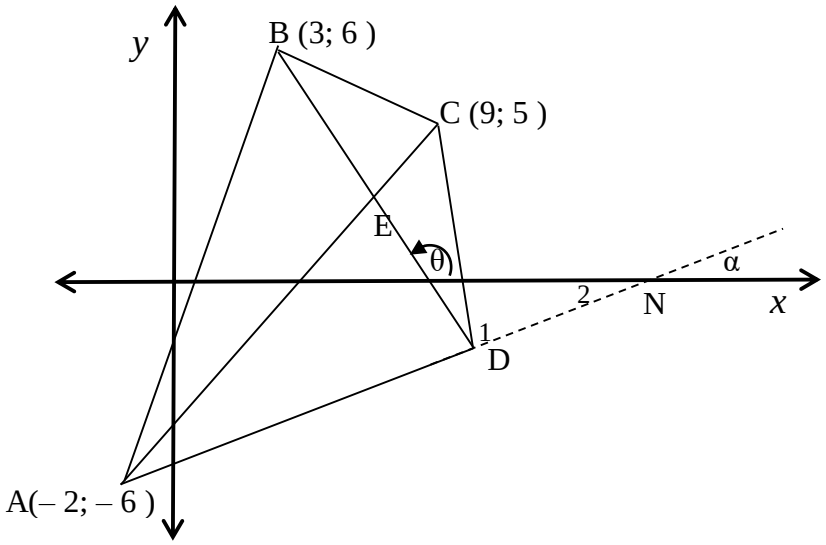
- **If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.*/Indien `n kandidaat `n poging doodgetrek het en NIE orgedoen het nie, word die doodgetrekte poging gemerk.***

- **Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error./*Konsekwente akkuraatheid word dwarsdeur die memorandum toegepas. Met die tweede berekeningsfout word nasien gestop.***

- **Assuming answers or values in order to solve a problem is NOT acceptable./**
  Die aanvaarding van antwoorde of waardes om `n problem op te los word NIE aanvaar nie./

GEOMETRY	
S	**A mark for a correct statement /*`n Punt vir die korrekte bewering.*** **(A statement mark is independent of a reason/** ***Hierdie punt is onafhanlik van die rede.)***
R	**A mark for a correct reason. /*`n Punt vir die korrekte rede.*** **(A reason mark may only be awarded if the statement is correct /*Hierdie punt word net toegeken as die berwering korrek is.)***
S/R	**Award a mark if statement AND reason are both correct/** ***Ken die punt toe as die bewering SOWEL AS die rede korrek is.***

QUESTION 1/*VRAAG 1*



1.1	$AC = \sqrt{(5+6)^2 + (9+2)^2}$ $AC = 11\sqrt{2}$	✓substitution/*vervanging* ✓ answer/*antwoord* (2)
1.2	$m_{AC} = \frac{5+6}{9+2}$ $m_{AC} = 1$	✓ substitution/*vervanging* ✓ $m_{AC} = 1$ (2)
1.3	$y - y_1 = m(x - x_1)$ C (9,5) or A (– 2; – 6) $y - 5 = x - 9$ $y + 6 = x + 2$ $y = x - 4$ $y = x - 4$	✓ substituting point A or C /*vervang punt A of C* ✓ answer/*antwoord* (2)
1.4	Diagonals of a kite intersect at right angles/ *Hoeklyne van vlieër sny loodreg/* So BE ⊥ AC.	✓ answer/*antwoord* (1)
1.5	$m_{BD} = -1$ B (3; 6) $y - 6 = -1(x - 3)$ $y = -x + 9$	✓ $m_{BD} = -1$ ✓ substitution/*vervanging* ✓ answer/*antwoord* (3)

1.6	At E, $x-4=-x+9$ $2x=13$ $x=6,5$ $y=2,5$ E (6,5; 2,5)	✓ $x-4=-x+9$ ✓ $x=6,5$ ✓ $y=2,5$ (3)
1.7	D(10;– 1) [AC symmetry line of kite/*AC lyn van simmetrie van vlieër, so AB = BD]* Answer only: 3/3	✓ $x=10$ ✓ $y=-1$ ✓ motivation/*motivering* (3)
1.8	$\tan\alpha=m_{AD}=\frac{5}{12}$ $\alpha=22,62^\circ$ $\hat{N}_2=\alpha=22,62^\circ$ $\tan\theta=m_{CD}=-6$ $\theta=99,46^\circ$ $\hat{D}_1=76,84^\circ$ [*ext* $\angle$ *of* Δ*/buite* $\angle$ *van* Δ] $A\hat{D}C=103,16^\circ$ [$\angle$*s on a str line*/reguit lyn/] **OR/*OF*** AB = 13, BC = $\sqrt{37}$ and/*en* $AC=11\sqrt{2}$ $AC^2=AB^2+BC^2-2AB\times BC\cos A\hat{B}C$ $(11\sqrt{2})^2=13^2+(\sqrt{37})^2-2(13)(\sqrt{37})\cos A\hat{B}C$ $\cos A\hat{B}C=-0,227...$ $A\hat{B}C=103,16^\circ$ $\therefore A\hat{D}C=103,16^\circ$ **OR/*OF*** $BE=\sqrt{3,5^2+(-3,5)^2}=\frac{7\sqrt{2}}{2}$ $\cos A\hat{D}E=\frac{3,5\sqrt{2}}{13}$ $A\hat{D}E=67,62^\circ$ $\cos C\hat{D}E=\frac{3,5\sqrt{2}}{\sqrt{37}}$ $C\hat{D}E=35,54^\circ$ $A\hat{D}C=103,16^\circ$	✓ $\alpha=22,62^\circ$ ✓ $\hat{N}_2=22,62^\circ$ ✓ $\theta=99,46^\circ$ ✓ $\hat{D}_1=76,84^\circ$ ✓ $A\hat{D}C=103,16^\circ$ ✓ correct lengths/*korrekte lengtes* ✓ substitution/*vervanging* ✓ $\cos A\hat{B}C=-0,227...$ ✓ $A\hat{B}C=103,16^\circ$ ✓ answer/*antwoord* ✓ $BE=\frac{7\sqrt{2}}{2}$ ✓ $\cos A\hat{D}E=\frac{3,5\sqrt{2}}{13}$ ✓ $A\hat{D}E=67,62^\circ$ ✓ $C\hat{D}E=35,54^\circ$ ✓answer/*antwoord* (5)

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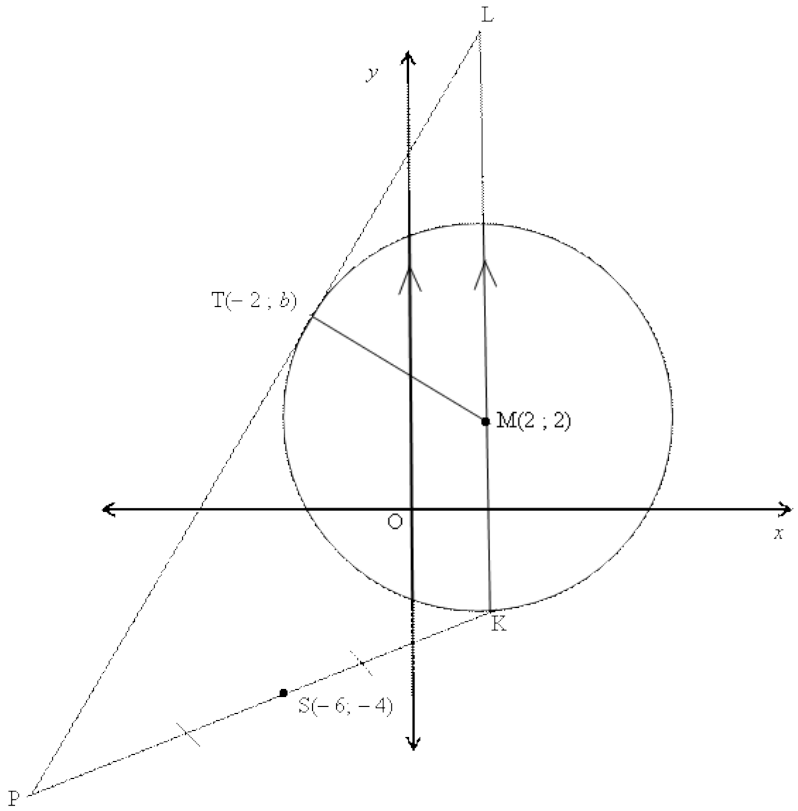
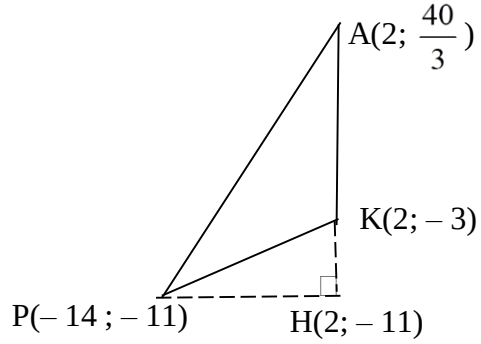
QUESTION 2/*VRAAG 2*



2.1	$(x-2)^2+(y-2)^2=25$ $(-2-2)^2+(b-2)^2=25$ $(b-2)^2=9$ or $16+b^2-4b+4=25$ $b-2=\pm 3$ $b^2-4b-5=0$ $(b-5)(b+1)=0$ $b=5$ or $b\neq -1$	✓ equation of the circle/ *vergelyking van sirkel* ✓substituting point T/ vervang T ✓ simplification/ vereenvoudiging Factors /*Faktore* ✓ answer /antwoord/ (4)
2.2.1	K(2; 2 – 5) K(2; – 3)	✓ $x=2$ ✓ $y=-3$ (2)
2.2.2	$m_{MT}=\frac{5-2}{-2-2}=-\frac{3}{4}$ $\therefore m_{PL}=\frac{4}{3}$ $y-y_1=m(x-x_1)$ $y-5=\frac{4}{3}(x+2)$ $y=\frac{4}{3}x+\frac{23}{3}$ **OR**	✓ $m_{MT}=-\frac{3}{4}$ ✓ $m_{PL}=\frac{4}{3}$ ✓ substitution /vervanging/ ✓ antwoord/*answer*

	$m_{PL} = \frac{5+11}{-2+14}$ $m_{PL} = \frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{23}{3}$	✓ substitution/ vervanging ✓ $m_{PL} = \frac{4}{3}$ ✓ substituting P or T/ *vervang P of T* ✓ answer/ antwoord (4)
2.2.3	 $y = \frac{4}{3}x + \frac{23}{3}$ $y = \frac{4}{3}(2) + \frac{23}{3}$ $y = \frac{31}{3}$ LK = $\frac{31}{3} + 3 = \frac{40}{3}$ Koordinate van P/*Coordinates of P* $\frac{x+2}{2} = -6$ $\frac{y-3}{2} = -4$ $x = -14$ $y = -5$ PH –hoogte/*Height* = 2 – (– 14) = 16 Area of PKL = $\frac{1}{2} \times \frac{40}{3} \times 16$ = $\frac{320}{3}$/106,67 vk eenhede/*sq units*	✓ $y_L = \frac{31}{3}$ ✓ LK = $\frac{40}{3}$ ✓ $x = -14$ ✓ $y = -11$ ✓ PH = 18 ✓ substitution/*vervanging* ✓ answer/*antwoord* (7)
2.3	The centres of the two circles lie on the same vertical line $x = 2$ and the sum of the radii is 10/ *Die middelpunte van die twee sirkels lê op dieselfde vertikale lyn* $x = 2$ *en die som van die radius = 10* $n - 2 = 10$ of/*or* $2 - n = 10$ $n = 12$ $n = -8$ Answer only: full mark/ Slegs antwoord: vol punte	✓ correct method/*korrekte metode* ✓ sum of radii = 10/ *som van* ri = 10 ✓ $n = 12$ ✓ $n = -8$ (4)

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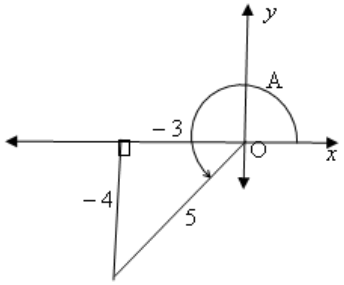
QUESTION 3/*VRAAG 3*

3.1	$\frac{\cos(90^\circ + x).\sin(540^\circ + x)}{\tan(x - 180^\circ).\cos(-x)}$ $= \frac{-\sin x.\sin(180^0 + x)}{\tan x.\cos x}$ $= \frac{-\sin x.(-\sin x)}{\frac{\sin x}{\cos x}.\cos x}$ $= \sin x$	✓ $-\sin x$ ✓ $\tan x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer/*antwoord* (6)
3.2	$\cos(P+Q) = \cos P.\cos Q - \sin P.\sin Q$ $\cos(2P) = \cos(P+P) = \cos P.\cos P - \sin P.\sin P$ $= \cos P.\cos P - \sin P.\sin P$ $= \cos^2 P - \sin^2 P$ $= \cos^2 P - (1 - \cos^2 P)$ $= \cos^2 P - 1 + \cos^2 P$ $= 2\cos^2 P - 1$	✓ $\cos 2P = \cos P.\cos P - \sin P.\sin P$ ✓ $\cos^2 P - \sin^2 P$ ✓ $1 - \cos^2 P$ ✓ $2\cos^2 P - 1$ (4)
3.3.1	$\tan A = \frac{4}{3}$, $A \in [90^\circ; 360^\circ]$  $\sin A = -\frac{4}{5}$	✓ r = 5 ✓ correct sketch and quad/ *korrekte skets en kwadrant* ✓ answer/*antwoord* (3)
3.3.2	$\cos(A + 30^\circ) = \cos A.\cos 30^\circ - \sin A \sin 30^\circ$ $= -\frac{3}{5}\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{4}{5}\right)\left(\frac{1}{2}\right)$ $= \frac{4 - 3\sqrt{3}}{10}$ OR/*OF* $-0{,}12$	✓ expansion/*uitbreiding* ✓ $\frac{3}{5}\left(\frac{\sqrt{3}}{2}\right)$ ✓ $\left(-\frac{4}{5}\right)\left(\frac{1}{2}\right)$ ✓ answer/*antwoord* (4)

3.4.1	$\sin 78^\circ = \cos 12^\circ$ $= p$	✓ $\cos 12^\circ$ ✓ answer/*antwoord* (2)
3.4.2	$\cos 12^\circ = \cos 2(6^\circ)$ $p = 1 - 2\sin^2 6^0$ $p - 1 = -2\sin^2 6^0$ $\frac{1-p}{2} = \sin^2 6^0$ $\sin 6^0 = \pm\sqrt{\frac{1-p}{2}}$	✓ $\cos 2(6^\circ)$ ✓ $p = 1 - 2\sin^2 6^\circ$ ✓ $\frac{1-p}{2} = \sin^2 6^\circ$ ✓ answer/*antwoord* (4)

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QUESTION 4/ *VRAAG 4*

4.1	$\frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$ LHS $= \frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1}$ $= \frac{2\sin x\cos x - (\cos^2 x - \sin^2 x) + 1}{2\sin x\cos x + (\cos^2 x - \sin^2 x) + 1}$ $= \frac{2\sin x\cos x - \cos^2 x + \sin^2 x + 1}{2\sin x\cos x + \cos^2 x - \sin^2 x + 1}$ $= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x}$ $= \frac{2\sin x(\cos x + \sin x)}{2\cos x(\sin x + \cos x)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ LHS = RHS	Both numerator and denominator/ *Sowel teller as noemer* ✓ $2\sin x\cos x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin x\cos x + 2\sin^2 x$ ✓ $2\sin x\cos x + 2\cos^2 x$ ✓ $2\sin x(\cos x + \sin x)$ ✓ $2\sin x(\sin x + \cos x)$ ✓ $\frac{\sin x}{\cos x}$ (7)

4.2	$\tan 22{,}5^\circ = \dfrac{\sin 45^\circ - \cos 45^\circ + 1}{\sin 45^\circ + \cos 45^\circ + 1}$ $= \dfrac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + 1}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1}$ $= \dfrac{1}{\sqrt{2}+1}$ or $\sqrt{2}-1$	✓ expansion/ *uitbreiding* ✓ substitution/*vervanging* ✓ answer/*antwoord* (3)
4.3	$3\cos 2x = 1 + 5\cos x$ $3(2\cos^2 x - 1) = 1 + 5\cos x$ $6\cos^2 x - 3 = 1 + 5\cos x$ $6\cos^2 x - 5\cos x - 4 = 0$ $(3\cos x - 4)(2\cos x + 1) = 0$ $\cos x = \dfrac{4}{3}$ or $\cos x = -\dfrac{1}{2}$ No solution $x = 120^\circ + k.360^\circ, k \in Z$ $x = 240^\circ + k.360^\circ, k \in Z$ **OR/*OF*** $x = \pm\cos^{-1}\left(-\dfrac{1}{2}\right) + k.360^\circ$ $x = \pm 120^\circ + k.360^\circ, k \in Z$	✓ $2\cos^2 x - 1$ ✓ std form/*std form* ✓ factors/*faktore* ✓ both equations/*beide vergelykings* ✓ No solution/*geen oplossing* ✓ Gen Solution 1/*alg oplossing1* ✓ Gen Solution 2/ *alg oplossing2/* – 1 as/*if* $k \in Z$ is not included *nie aangedui* (7)

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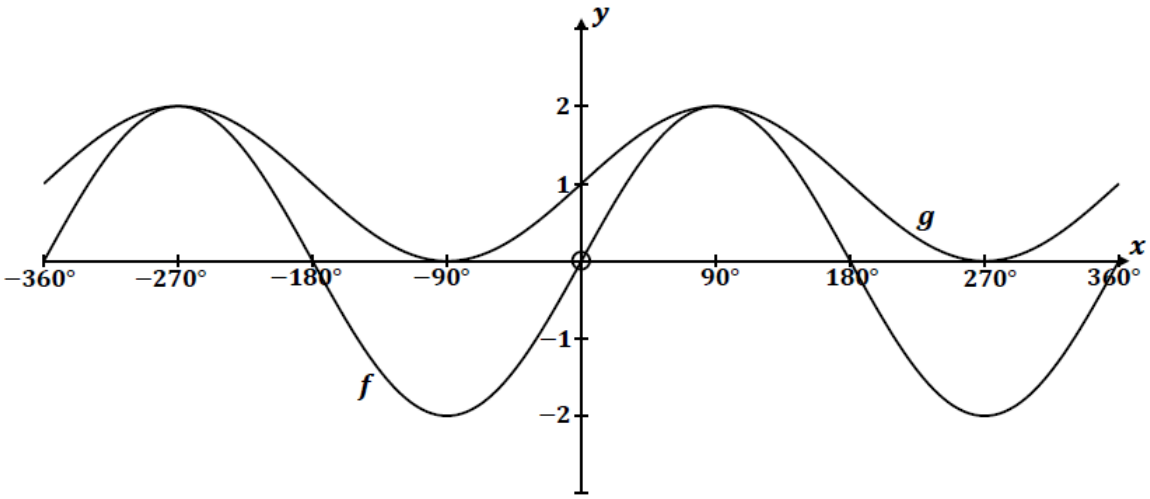
QUESTION 5/*VRAAG 5*

$f(x) = a\sin x$ en/*and* $g(x) = \sin x + b$; $x \in [-360^\circ; 360^\circ]$.



5.1	$a = 2$ $b = 1$	✓ $a = 2$ ✓ $b = 1$ (2)
5,2	Amplitude = 1	✓ answer/*antwoord* (1)
5.3.1	$x = -270^\circ$ of/*or* $x = 90^\circ$	✓ $x = -270^\circ$ ✓ $x = 90^\circ$ (2)
5.3.2	$-360^\circ < x < -270^\circ$ or $-90^\circ < x < 90^\circ$ or $270^\circ < x < 360^\circ$ OF/ *OR* $x \in (-360^\circ; -270^\circ)$ or $x \in (-90^\circ; 90^\circ)$ or $x \in (270^\circ; 360^\circ)$	✓ $-360^\circ < x < -270^\circ$ ✓ $-90^\circ < x < 90^\circ$ ✓ $270^\circ < x < 360^\circ$ OF/*OR* ✓ $x \in (-360^\circ; -270^\circ)$ ✓ $x \in (-90^\circ; 90^\circ)$ ✓ $x \in (270^\circ; 360^\circ)$ (3)
5.3.3	$-270^\circ < x < -90^\circ$ or $90^\circ < x < 270^\circ$ $x \in (-270^\circ; -90^\circ)$ or $x \in (90^\circ; 270^\circ)$	✓ $-270^\circ < x < -90^\circ$ ✓ $90^\circ < x < 270^\circ$ OF/*OR* ✓ $x \in (-270^\circ; -90^\circ)$ ✓ $x \in (90^\circ; 270^\circ)$ (2)

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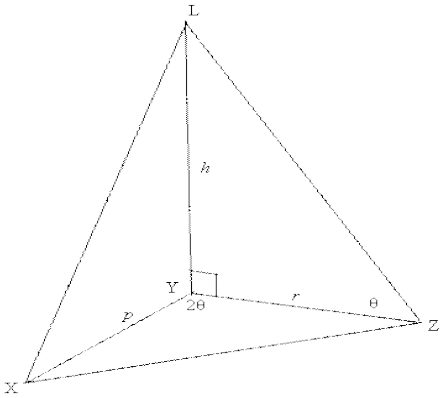
QUESTION 6/*VRAAG 6*



6.1	$A = \frac{1}{2} \times p \times r \times \sin 2\theta$	✓ answer/antwoord (1)
6.2	$A = \frac{1}{2} \times p \times r \times \sin 2\theta$ $r = \frac{A}{\frac{1}{2} p \sin 2\theta}$ $r = \frac{2A}{p \sin 2\theta}$	✓ $r = \frac{A}{\frac{1}{2} p \sin 2\theta}$ ✓ $r = \frac{2A}{p \sin 2\theta}$ (2)
6.3	$\tan\theta = \frac{h}{r}$ $h = r \tan\theta$ $h = \frac{2A}{p \sin 2\theta} \times \frac{\sin\theta}{\cos\theta}$ $h = \frac{2A}{p \times 2\sin\theta\cos\theta} \times \frac{\sin\theta}{\cos\theta}$ $h = \frac{A}{p\cos^2\theta}$	✓ $\tan\theta = \frac{h}{r}$ ✓ $h = r\tan\theta$ ✓ $h = \frac{2A}{p \sin 2\theta} \times \frac{\sin\theta}{\cos\theta}$ ✓ $2\sin\theta\cos\theta$ ✓ answer/antwoord (5)
6.4.1	$h = \frac{A}{p\cos^2\theta}$ $20 = \frac{A}{10\cos^2 60^\circ}$ $A = 50\ m^2$	✓ substitution/ vervanging ✓ answer/antwoord (2)
6.4.2	$r = \frac{2A}{p \sin 2\theta}$ $r = \frac{2(50)}{10 \sin 2(60^\circ)}$ $r = 11{,}55\ m$ $t^2 = 10^2 + (11{,}55)^2 - 2(10)(11{,}55)\cos 2(60^\circ)$ $t = 18{,}68\ m$	✓ substitution/ vervanging ✓ $r = 11{,}55\ m$ ✓ correct substitution korrekte vervanging ✓ substitution/vervanging (4)

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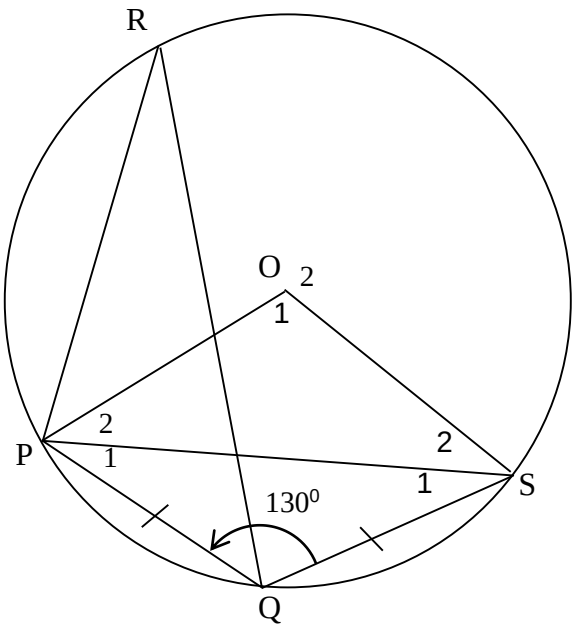
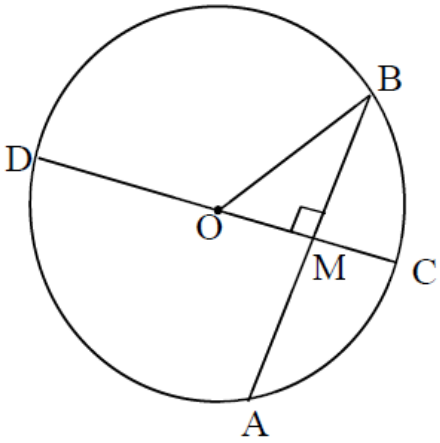
QUESTION 7/*VRAAG 7*



7.1.1	$\hat{S}_1 = \hat{P}_1$ [∠`e teenoor = sye/∠*s opp = sides*] $2\hat{S}_1 + 130^\circ = 180^\circ$ [binne ∠`e van Δ/∠*s of a Δ*] $\hat{S}_1 = 25^\circ$	✓ S/R ✓ S/R ✓ answer/antwoord (3)
7.1.2	$\hat{R} = \hat{S}_1 = 25^\circ$ [omtrekshoeke/∠*s subt by same chord]*	✓ S ✓ R (2)
7.1.3	$\hat{O}_2 = 260^\circ$ [mdpt ∠=2x omtreks∠/*angle at centre = 2∠ at circumf]* $\hat{O}_1 = 100^\circ$ [omwenteling/∠*s around a point*]	✓ S ✓ R ✓ answer/antwoord (3)

7.2		
7.2.1	DC = $9x$	✓ answer/antwoord (1)
7.2.2	OM = OC – MC $OM = \frac{9x}{2} - 2x$ $OM = 2{,}5x$	✓ method/*metode* ✓ answer/antwoord (2)
7.2.3	$OB^2 = BM^2 + OM^2$ BM = 7 [line from centre ⊥ to chord/*radius* ⊥ *koord*] $\left(\frac{9x}{2}\right)^2 = 7^2 + \left(\frac{5x}{2}\right)^2$ $\frac{81x^2}{4} = 49 + \frac{25x^2}{4}$ $14x^2 = 49$ $x^2 = 3{,}5$ $x = 1{,}87$ radius = $\frac{9}{2}(1{,}87) = 8.42$ units	✓ S/R ✓ substitution/vervanging ✓ $x^2 = 3{,}5$ ✓ $x = 1{,}87$ ✓ radius = 8,42 eenhede/*units* (5)

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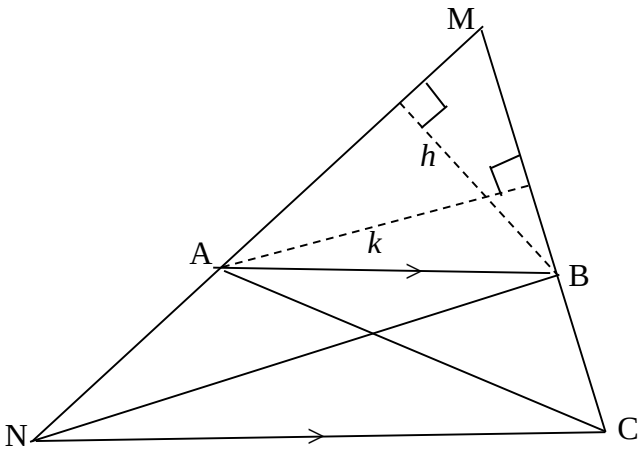
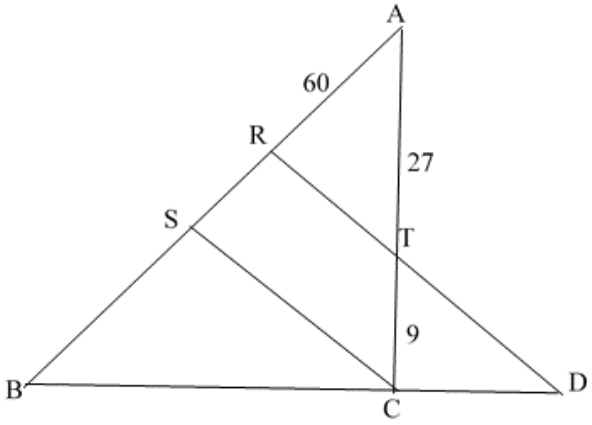
QUESTION 8/*VRAAG 8*



8.1	Const: Draw height k from A such that $k \perp$ MB and h from B such that $h \perp$ AM. *Konstr: Trek hoogte k vanaf A sodat $k \perp MB$ en h vanaf B sodat $h \perp AM$.* Proof/Bewys:*:* $\frac{\text{area } \Delta MAB}{\text{area } \Delta ANB} = \frac{\frac{1}{2}MA \times h}{\frac{1}{2}AN \times h}$ $\frac{\text{area } \Delta MAB}{\text{area } \Delta ANB} = \frac{AM}{AN}$ $\frac{\text{area } \Delta MAB}{\text{area } \Delta ABC} = \frac{\frac{1}{2}MB \times k}{\frac{1}{2}BC \times k} = \frac{MB}{BC}$ Maar/*But* area ΔANB = area ΔABC [same base and height/ *selfde basis selfde hoogte* of/*or* RP // TM] $\frac{MA}{AN} = \frac{MB}{BC}$	✓ construction/konstruksie ✓ Area $\Delta MAB = \frac{1}{2}MA \times h$ ✓ Area $\Delta ANB = \frac{1}{2}AN \times h$ ✓ $\frac{\text{area } \Delta MAB}{\text{area } \Delta ANB} = \frac{AM}{AN}$ ✓ $\frac{\text{area } \Delta MAB}{\text{area } \Delta ABC} = \frac{MB}{BC}$ ✓ S ✓ R (7)



8.2.1	$\frac{AC}{TA}=\frac{36}{27}=\frac{4}{3}$ $\frac{AS}{AR}=\frac{80}{60}=\frac{4}{3}$ $\frac{AC}{TA}=\frac{AS}{AR}$ SC || RD [converse line || to one side of Δ/ *omgekeerd lyn || een sy van* Δ OR Converse prop theorem, SC || RD/ *Omgekeerd eweredigheid stelling* SC || RD OR lyn verdeel sy eweredig	✓ $\frac{AC}{TA}=\frac{4}{3}$ ✓ $\frac{AS}{AR}=\frac{4}{3}$ ✓ R (3
8.2.2	RS = 80 – 60 = 20 cm	✓ answer/*antwoord* (1)
8.2.3	$RB=\frac{3}{2}AR$ $RB=\frac{3}{2}(60)$ RB = 90cm SB = 70 cm $\frac{CD}{30}=\frac{20}{70}$ [line || to one side of Δ/*lyn || een sy van* Δ **OR/OF** [prop theorem, SC || RD] **OR/*OF*** *omgekeerd eweredigheid stelling* SC || RD] CD = 8,57 cm	✓ RB = 90 cm ✓ SB = 70 cm ✓ S ✓ R ✓ answer/*antwoord* (5)

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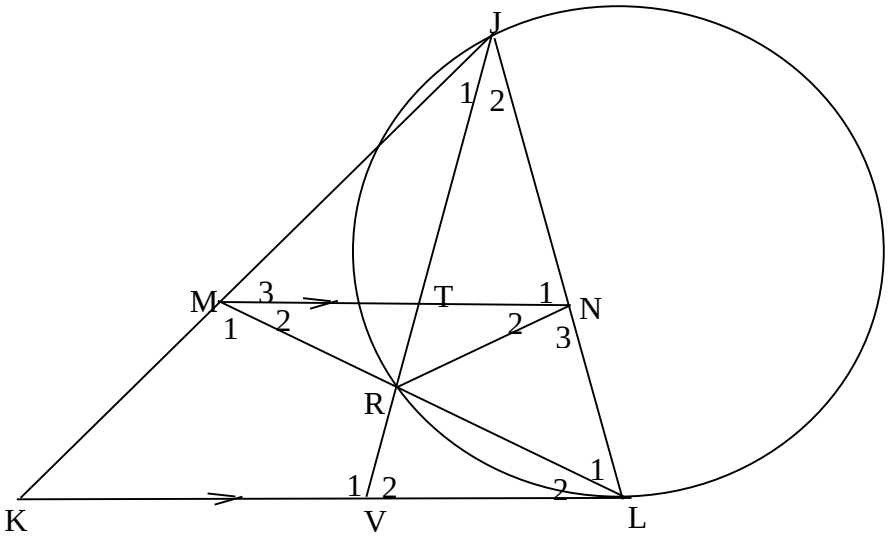
QUESTION 9/*VRAAG 9*



9.1	$\hat{L}_2 = \hat{M}_2$ [alt ∠s/verwiss ∠`e/ MN || KL] $\hat{J}_2 = \hat{L}_2$ [tan chord theorem/raaklyn-koord stelling*]* ∴JMRN is a cyclic quad/ *is `n koordevierhoek* [converse of ∠s in same seg/ *omgekeerde omtrekshoeke*]	✓ S/R ✓ S /R ✓ R (3)
9.2	In ΔJNR and/*en* ΔLMK $\hat{J}_2 = \hat{L}_2$ [tan chord theorem/raaklyn-koord stelling] $J\hat{N}R = \hat{M}_1$ [ext ∠ of cyclic quad/buitehoek van kvh] $\hat{R}_2 = \hat{K}$ [∠s of a Δ/binnehoeke van Δ] ∴ΔJNR ||| ΔLMK [AAA/*HHH*]	✓ S/R ✓ S/R ✓ 3rd ∠ of Δ/*3de ∠ van* Δ or/*of R* (3)
9.3	$\frac{JT}{TV} = \frac{JN}{NL}$ [Prop theorem/*Eweredigheid stelling*, , TN || VL OF/*OR* Line || to one side of Δ/*Lyn || een sy van Δ*] $JN = \frac{NL.JT}{TV}$ $\frac{JN}{LM} = \frac{NR}{MK}$ [ΔJNR ||| ΔLMK] $JN = \frac{LM.NR}{MK}$ $\therefore \frac{NL.JT}{TV} = \frac{LM.NR}{MK}$	✓ S ✓ R ✓ $JN = \frac{NL.JT}{TV}$ ✓ S ✓ R ✓ $JN = \frac{LM.NR}{MK}$ (6)

[12]

TOTAAL/*TOTAL*: 150