



Education and Sport Development

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

JUNE 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 2 pages of formulas, an answer sheet and 2 diagram sheets.



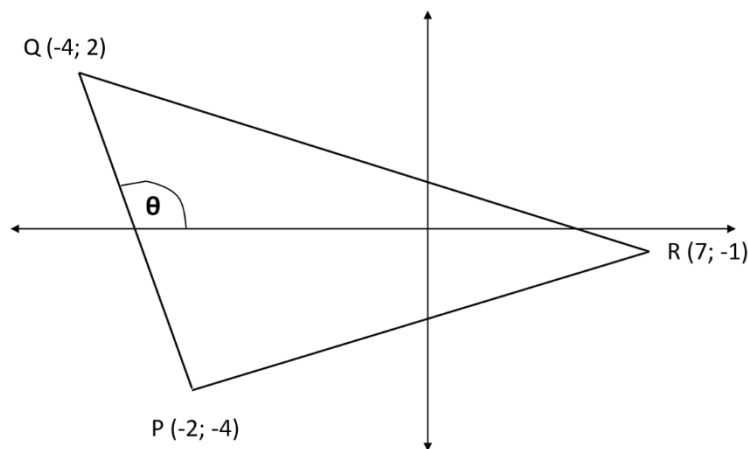
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this question paper.
10. An answer sheet for Question 5.1 is attached at the end of this question paper.
11. A diagram sheet with the geometry diagrams for Question 7 and Question 8 are attached at the end of this question paper.
12. Write neatly and legibly.

QUESTION 1

$P(-2; -4)$; $Q(-4; 2)$ and $R(7; -1)$ are the corners of ΔPQR on a cartesian plane.

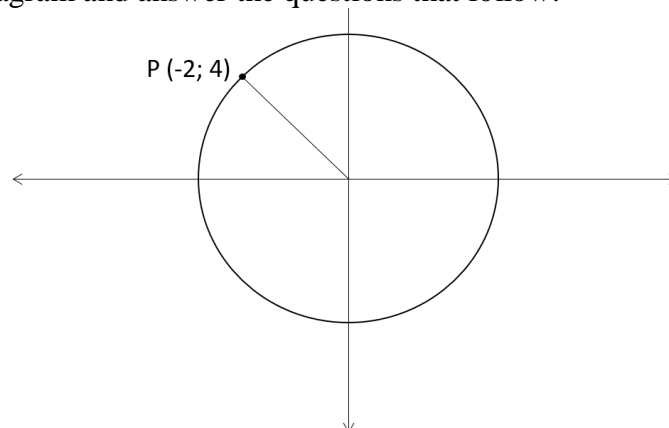


Determine:

- 1.1 The gradient of line PQ . (2)
 - 1.2 The gradient of line PR . (2)
 - 1.3 What type of triangle is ΔPQR . (1)
 - 1.4 Determine the coordinates of M the midpoint of PQ . (3)
 - 1.5 Determine the size of θ the angle of inclination of PQ . (4)
- [12]**

QUESTION 2

Study the following diagram and answer the questions that follow:

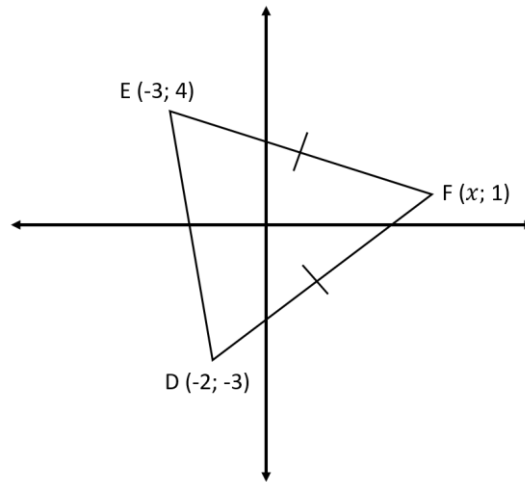


- 2.1 Determine the equation of the circle in the diagram with its centre at the origin. (2)
 - 2.2 Determine the equation of the tangent line to the circle at point P . (5)
 - 2.3 Sketch the ellipse with the equation $9x^2 + 4y^2 = 36$ (3)
- [10]**



QUESTION 3

Given $\triangle DEF$ with vertices $D(-2; -3)$, $E(-3; -4)$ and $F(x; 1)$. It is also given that $DF=EF$.

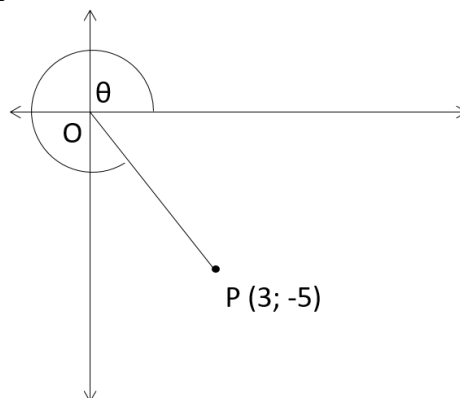


Determine the value of x .

(7)
[7]

QUESTION 4

4.1 In the diagram below P is the point $(3; -5)$ on the Cartesian Plane with Origin O . Use the diagram to answer the questions below:



4.1.1 Determine the length of OP . (2)

4.1.2 Determine, without the use of a calculator, the value(s) of:

(a) $\sin \theta$ (1)

(b) $1 + \cot^2 \theta$ (2)

4.2 Complete the following trigonometric identities:

4.2.1 $1 - \sin^2 \beta =$ (1)

4.2.2 $\tan^2 \beta + 1 =$ (1)



4.3 Use the trigonometric identities to prove the following equations:

$$4.3.1 \quad \cos^2 \theta + \sin^2 \theta + \tan^2 \theta = \sec^2 \theta \quad (2)$$

$$4.3.2 \quad \sin^2 \theta + \sin^2 \theta \cot^2 \theta = 1 \quad (4)$$

4.4 Simplify without the use of a calculator:

$$\frac{\sin(180^\circ + x) \cdot \cos 330^\circ \cdot \tan 150^\circ}{\sin x} \quad (7)$$

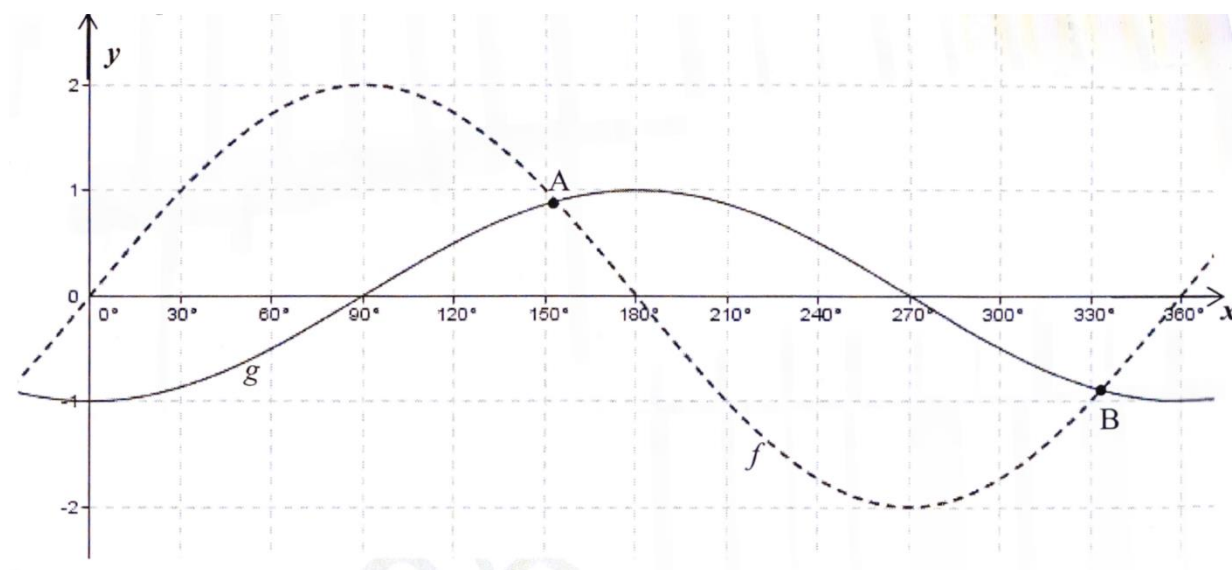
4.5 Determine the value of θ for $\theta \in [90^\circ; 270^\circ]$ if $3 \cos \theta + 2 = 0$ (4)

[24]

QUESTION 5

5.1 On the same set of axes, sketch the graphs defined by $f(x) = \tan x$ and $g(x) = \sin 2x$ for the domain $x \in [0^\circ; 270^\circ]$. Clearly label your graphs and show all salient points (6)

5.2 Given the graphs of $f(x) = a \sin x$ and $g(x) = b \cos x$ for $x \in [0^\circ; 360^\circ]$



Use the given graphs to answer the following questions:

5.2.1 Determine the numerical value of

(a) a (1)

(b) b (1)

5.2.2 Write down the range of f . (1)

5.2.3 Write down the value of x where $f(x) = -2$. (1)

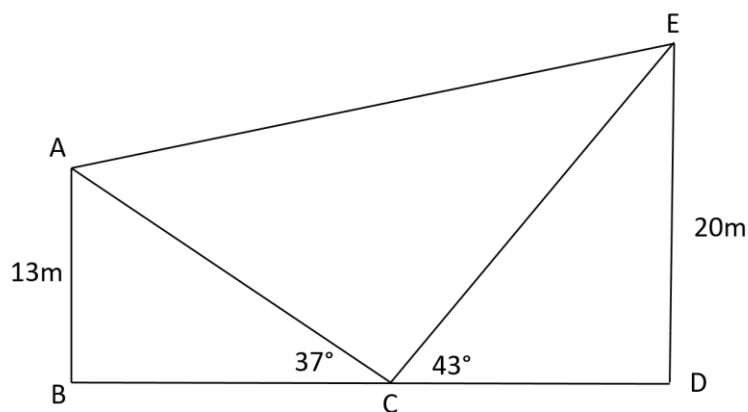


5.2.4 Points A and B are exactly out of phase. If the co-ordinates of A are $(153.4^\circ; 0.89)$, write down the co-ordinates of B. (2)

5.2.5 For what value(s) of x is $g(x) > f(x)$? (2)
[14]

QUESTION 6

AB and DE are two towers on the same horizontal plane. The angle of elevation from point C to point E, the top of tower DE, is 43° . The angle of elevation from point C to point A, the top of tower AB is 37° . Tower AB has a height of 13 m and tower DE a height of 20 m.



6.1 Complete the following formulas for any triangle MNO:

6.1.1 $\frac{\sin M}{\dots} = \frac{m}{n}$ (1)

6.1.2 $m^2 = n^2 + o^2 \dots$ (1)

6.2 Determine the length of AC. (2)

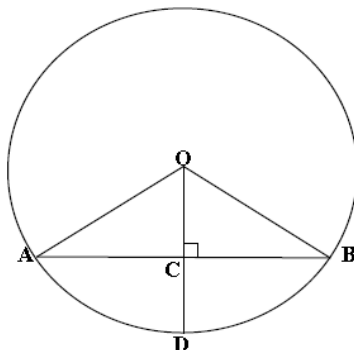
6.3 If CE is 29.33 m, determine AE. (5)

6.4 Determine the area of $\triangle ACE$. (3)
[12]



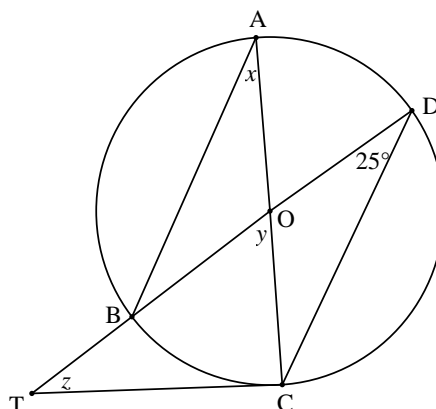
QUESTION 7

- 7.1 In the diagram below, AB is a chord of the circle with center O. D is a point on the circle. OD is perpendicular on AB. OA = 25 cm and CD = 18 cm.



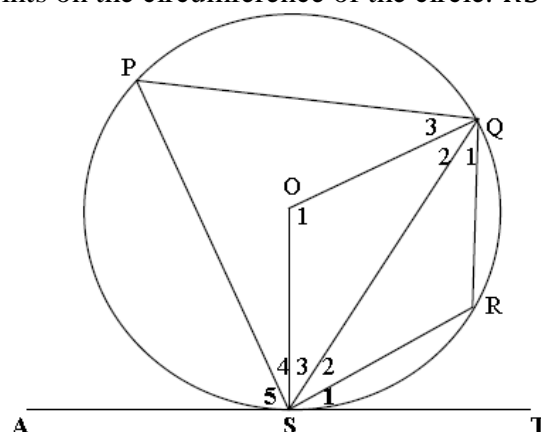
Determine with reasons the length of AB. (5)

- 7.2 O is the center of the circle. TC is a tangent to the circle at the point C. AC and DB are diameters of the circle and DB is extended to point T. $\widehat{D} = 25^\circ$.



Determine with reasons the size of x , y and z . (5)

- 7.3 A circle with center O is shown in the diagram below. AST is a tangent to the circle at S. P, Q, R and S are points on the circumference of the circle. $\widehat{RST} = \widehat{S_1} = 23^\circ$ and $QR = RS$.



Determine with reasons the sizes of:

7.3.1 \widehat{S}_2 (4)

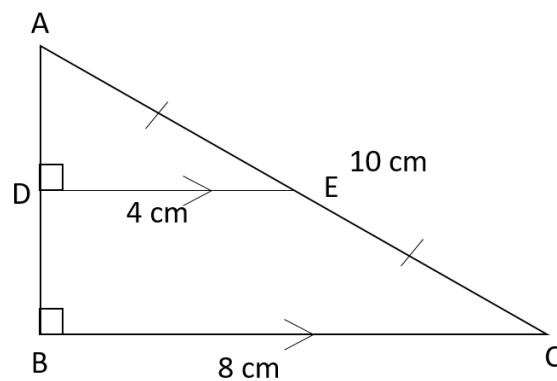
7.3.2 \widehat{R} (2)

7.3.3 \widehat{P} (2)

7.3.4 \widehat{O}_1 (2)
[20]

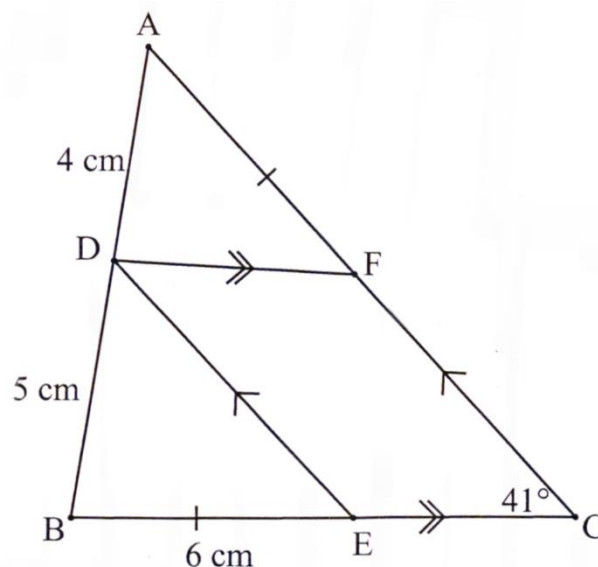
QUESTION 8

- 8.1 In the figure below angles \widehat{ADE} and \widehat{ABC} are right angles and $DE \parallel BC$. $AE = 5$ cm, $DE = 4$ cm and $BC = 8$ cm. $AC = 10$ cm and $AE = EC$.



Determine the length of DB. (4)

- 8.2 In the diagram ABC is a triangle with D, E and F on AB, BC and AC respectively. $AD = 4$ cm, $DB = 5$ cm and $BE = AF = 6$ cm. Quadrilateral DECF has $DE \parallel CF$ and $DF \parallel EC$. The size of \widehat{BCA} is 41° .



8.2.1 What type of quadrilateral is DECF? (1)

8.2.2 Determine with reasons:

(a) Three other angles equal to $\hat{BCA} = 41^\circ$ (6)

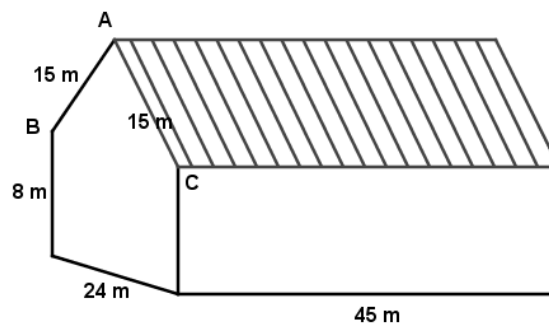
(b) EC. (4)

(c) FC. (3)

[18]

QUESTION 9

A farmer builds a shed with the dimensions shown below on his farm.



9.1 Determine the total area of the roof of the building. (3)

9.2 Determine the surface area of all the walls of the building. (5)

9.3 Determine the useable area of the floor. (1)

9.4 If the owner decides to install an attic. What would the volume of the storage space of the attic be? (3)

[12]

QUESTION 10

10.1 A car is taken for a break test at a vehicle testing station. The front wheel of the car is placed on two cylinders with equal radii to perform the test. The side view of the front wheel is shown below. The wheel of the car has a diameter of 80 cm.



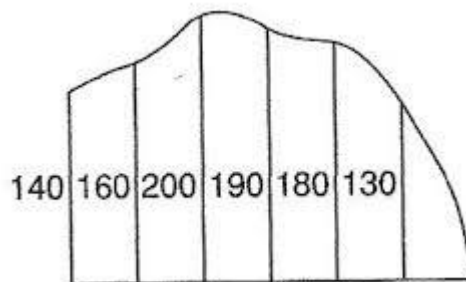
10.1.1 Determine the speed of the car (in km/h) when the wheel rotates at 420 revolutions per minute. (5)

10.1.2 Determine the radius of the small cylinders if they rotate at 1680 revolutions per minute. (5)

10.2 A sprinkler head can spray water over 15 meters. Calculate the total area the sprinkler can water if it the sprinkler head rotates 145° . (5)
[15]

QUESTION 11

The base of the irregular figure shown below is 300 units wide.



Use the mid-ordinate rule to calculate the area of the figure. (6)
[6]

TOTAL: 150



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area of } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$



$$\pi \text{ rad} = 180^\circ$$

$$\text{Angular velocity} = \omega = 2\pi n = 360^\circ n \quad \text{where } n = \text{rotation frequency}$$

$$\text{Circumferential velocity} = v = \pi D n \quad \text{where } D = \text{diameter and } n = \text{rotation frequency}$$

$$s = r\theta \quad \text{where } r = \text{radius and } \theta = \text{central angle in radians}$$

$$\text{Area of a sector} = \frac{rs}{2} = \frac{r^2\theta}{2} \quad \text{where } r = \text{radius, } s = \text{arc length and}$$

$$\theta = \text{central angle in radians}$$

$$4h^2 - 4dh + x^2 = 0 \quad \text{where } h = \text{height of segment, } d = \text{diameter of circle and}$$

$$x = \text{length of chord}$$

$$A_T = a(m_1 + m_2 + m_3 + \dots + m_n) \quad \text{where } a = \text{equal parts, } m_1 = \frac{o_1 + o_2}{2}$$

$$\text{and } n = \text{number of ordinates}$$

OR

$$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right) \quad \text{where } a = \text{equal parts, } o_i = i^{\text{th}} \text{ ordinate}$$

$$\text{and } n = \text{number of ordinates}$$



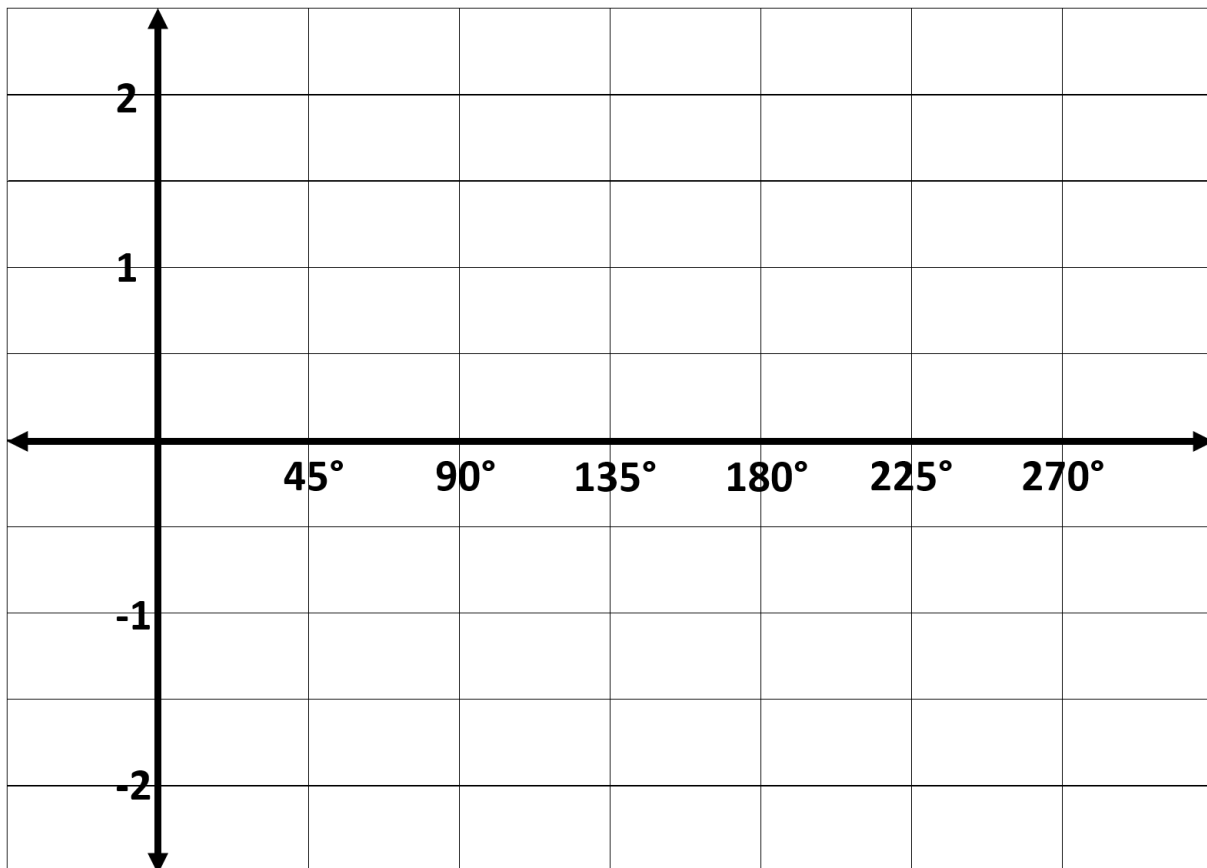
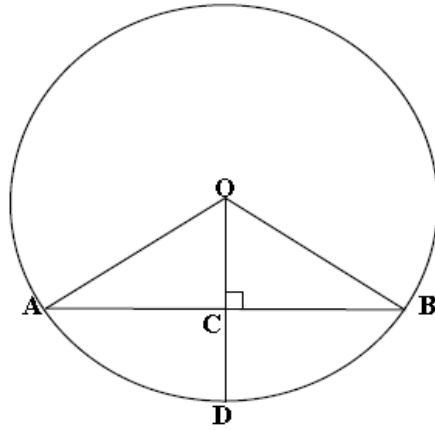
ANSWER SHEET / ANTWOORDBLAD**NAME:** _____**QUESTION 5.1 / VRAAG 5.1**

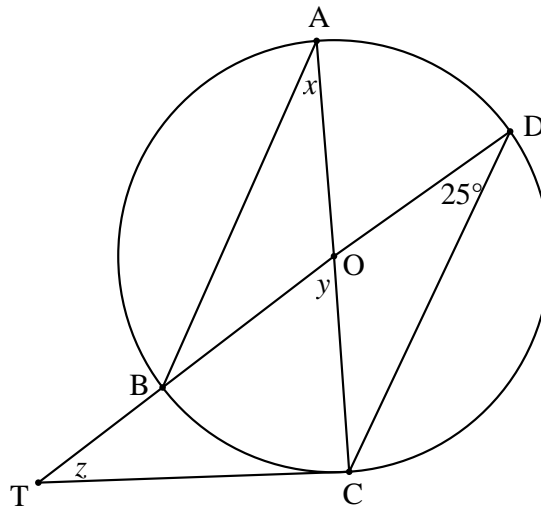
DIAGRAM SHEET / DIAGRAMVEL

QUESTION 7 / VRAAG 7

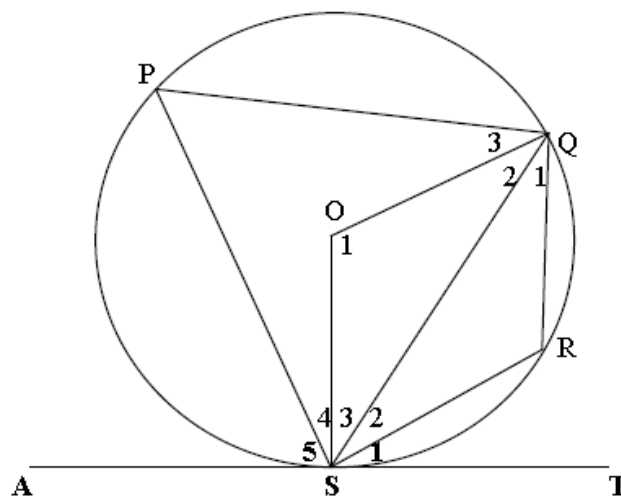
7.1



7.2

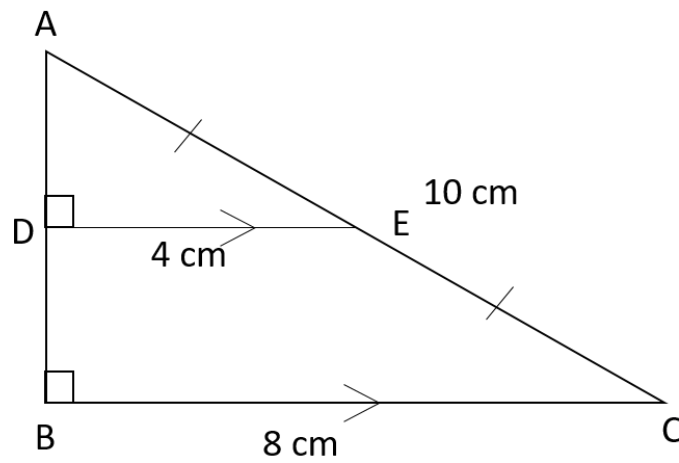


7.3



QUESTION 8 / VRAAG 8

8.1



8.2

