



Education and Sport Development

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

JUNE 2019

MEMORANDUM

MARKS: 150

This memorandum consists of 13 pages.

NOTE:

1. If a candidate answered a question TWICE, only mark the FIRST attempt.
2. If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
3. Consistent accuracy applies throughout in ALL aspects of the marking memorandum.

QUESTION 1

1.1	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4 - (2)}{-2 - (-4)}$ $= -3$ <p>OR</p> $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{2 - (-4)}{-4 - (-2)}$ $= -3$	✓ Substitution ✓ Answer (2)
1.2	$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4 - (-1)}{-2 - 7}$ $= \frac{1}{3}$ <p>OR</p> $m_{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-1 - (-4)}{7 - (-2)}$ $= \frac{1}{3}$	✓ Substitution ✓ Answer (2)
1.3	Right angled triangle	✓ Answer (1)

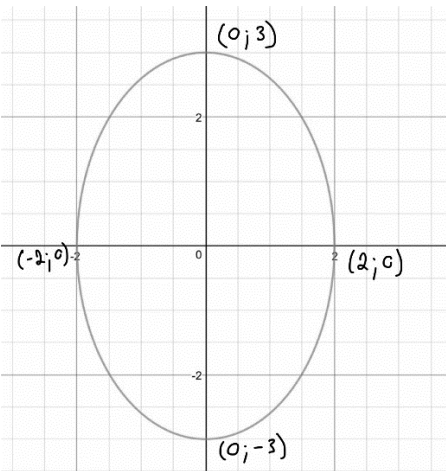


1.4	$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M\left(\frac{-2 + (-4)}{2}, \frac{-4 + 2}{2}\right)$ $M(-3; -1)$ <p>OR</p> $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M\left(\frac{-4 + (-2)}{2}, \frac{2 + (-4)}{2}\right)$ $M(-3; -1)$	✓ Formula ✓ Substitution ✓ Answer (3)
1.5	$\tan \theta = m$ $\tan \theta = -3$ $\text{ref } \angle = \tan^{-1}(3)$ $= 71.57$ $\theta = 180 - 71.57$ $= 108.43$	✓ Formula ✓ Substitution ✓ Ref Angle Answer ✓ Answer (4) [12]

QUESTION 2

2.1	$x^2 + y^2 = r^2$ $(-2)^2 + (4)^2 = r^2$ $20 = r^2$ $x^2 + y^2 = 20$	✓ Substitution ✓ Final Equation (2)
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2.2	$m_{OP} = \frac{4}{-2}$ $= -2$ $m_{\text{tangent}} = \frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - 4 = \frac{1}{2}(x - (-2))$ $y - 4 = \frac{1}{2}x + 1$ $y = \frac{1}{2}x + 5$ OR $y = mx + c$ $4 = \frac{1}{2}(-2) + c$ $4 = -1 + c$ $c = 5$ $y = \frac{1}{2}x + 5$	✓ m_{OP} ✓ m_{tangent} ✓ Substitution ✓ Simplify ✓ Equation (5) OR ✓ Substitution ✓ Simplify ✓ Equation (5)
2.3	$9x^2 + 4y^2 = 36$ $\frac{x^2}{4} + \frac{y^2}{9} = 1$ 	✓ Standard Form ✓ Both x intercepts ✓ Both y intercepts (3) [10]

QUESTION 3

3	$EF = \sqrt{(x - (-3))^2 + (1 - (4))^2}$ $= \sqrt{x^2 + 6x + 9 + 9}$ $= \sqrt{x^2 + 6x + 18}$ $DF = \sqrt{(x - (-2))^2 + (1 - (-3))^2}$ $= \sqrt{x^2 + 4x + 4 + 16}$ $= \sqrt{x^2 + 4x + 20}$ $DF = EF$ $x^2 + 6x + 18 = x^2 + 4x + 20$ $2x = 2$ $x = 1$	✓ Substitution ✓ EF Answer ✓ Substitution ✓ DF Answer ✓ Equations set = ✓ Simplify ✓ Answer [7]
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QUESTION 4

4.1.1	$OP^2 = x^2 + y^2$ $= (3)^2 + (-5)^2$ $= 34$ $OP = \sqrt{34}$	✓ Substitution ✓ Answer (2)
4.1.2 (a)	$\sin \theta = \frac{y}{r}$ $= \frac{-5}{\sqrt{34}}$	✓ Answer (1)
4.1.2 (b)	$1 + \cot^2 \theta = 1 + \left(\frac{3}{-5}\right)^2$ $= 1 + \frac{9}{25}$ $= \frac{34}{25}$	✓ Substitution ✓ Answer (2)
4.2.1	$\cos^2 \beta$	✓ Answer (1)



4.2.2	$\sec^2 \beta$	✓ Answer (1)
4.3.1	$\cos^2 \theta + \sin^2 \theta + \tan^2 \theta = 1 + \tan^2 \theta$ $= \sec^2 \theta$	✓ $\cos^2 \theta + \sin^2 \theta = 1$ ✓ Answer (2)
4.3.2	$\sin^2 \theta + \sin^2 \theta \cdot \cot^2 \theta = \sin^2 \theta (1 + \cot^2 \theta)$ $= \sin^2 \theta \cdot \operatorname{cosec}^2 \theta$ $= \sin^2 \theta \cdot \frac{1}{\sin^2 \theta}$ $= 1$	✓ Factorization ✓ Apply Identity ✓ Apply Identity ✓ Answer (4)
4.4	$\frac{\sin(180^\circ + x) \cdot \cos(360^\circ - 30^\circ) \cdot \tan(180^\circ - 30^\circ)}{\sin x}$ $= \frac{\sin x \cdot \cos 30^\circ \cdot -\tan 30^\circ}{\sin x}$ $= \cos 30^\circ \cdot \tan 30^\circ$ $= \sin 30^\circ$ $= \frac{1}{2}$	✓ $\cos(360^\circ - 30^\circ)$ ✓ $\tan(180^\circ - 30^\circ)$ ✓ $\cos 30^\circ$ ✓ $-\tan 30^\circ$ ✓ $\sin 30^\circ$ ✓ Answer (7)
4.5	$3 \cos \theta + 2 = 0$ $3 \cos \theta = -2$ $\cos \theta = \frac{-2}{3}$ $\operatorname{ref} \angle = \cos^{-1} \left(\frac{2}{3} \right)$ $= 48.19^\circ$ $\theta = 180^\circ + 48.19^\circ$ $= 228.19^\circ$ or $\theta = 180^\circ - 48.19^\circ$ $= 131.81^\circ$	✓ $\cos \theta = -\frac{2}{3}$ ✓ Ref Angle. ✓ Solution 1 ✓ Solution 2 (4) [24]

QUESTION 5

5.1		<p>Tan:</p> <ul style="list-style-type: none"> ✓ Shape ✓ x – intercepts ✓ Asymptotes <p>Sin:</p> <ul style="list-style-type: none"> ✓ Shape ✓ x – intercepts ✓ Turning points 	(6)
5.2.1 (a)	$a = 2$	✓ Answer	(1)
5.2.1 (b)	$b = -1$	✓ Answer	(1)
5.2.2	$-2 \leq y \leq 2$	✓ Answer	(1)
5.2.3	270°	✓ Answer	(1)
5.2.4	$(333.4^\circ; -0.89)$	<ul style="list-style-type: none"> ✓ x-value ✓ y-value 	(2)
5.2.5	$0 < x < 153.4^\circ$ and $x > 333.4^\circ$	<ul style="list-style-type: none"> ✓ Interval 1 ✓ Interval 2 	(2) [14]

QUESTION 6

6.1.1	$\sin N$	✓ Answer	(1)
6.1.2	$-2 \sin C \cos M$	✓ Answer	(1)
6.2	$\sin 37^\circ = \frac{13}{AC}$ $AC = \frac{13}{\sin 37^\circ}$ $= 21.6$	<ul style="list-style-type: none"> ✓ Sin relationship ✓ Answer 	(2)

6.3	$\hat{ACE} = 180^\circ - 37^\circ - 43^\circ$ $= 100^\circ$ $AE^2 = AC^2 + CE^2 - 2(AC)(CE)\cos C$ $= (21.6)^2 + (29.33)^2 - 2(21.6)(29.33)\cos 100^\circ$ $= 1546.83$ $AE = 39.33$	✓ $\hat{ACE} = 100$ ✓ Formula ✓ Substitution ✓ Simplify ✓ Answer (5)
6.4	$\text{Area} = \frac{1}{2}(AC)(CE)\sin C$ $= \frac{1}{2}(21.6)(29.33)\sin 100$ $= 311.95$	✓ Formula ✓ Substitution ✓ Answer (3) [12]

QUESTION 7

7.1	$OD = OA = 25\text{cm}$ Radius $OC = 25 - CD$ $= 25 - 18$ $= 7\text{cm}$ $OA^2 = AC^2 + OC^2$ Pythagoras $25^2 = AC^2 + 7^2$ $AC^2 = 576$ $AC = 24$ $AB = 48\text{m}$ Line from centre \perp to chord OR $d = 2OA$ Diameter = 2 x Radius $= 2(25)$ $= 50\text{cm}$ $4h^2 - 4dh + x^2 = 0$ Relation between $4(18)^2 - 4(50)(18) + x^2 = 0$ segment height, segment $1296 - 3600 + AB^2 = 0$ width and diameter $AB^2 = 2304$ $AB = 48\text{cm}$	✓ Statement ✓ Reason ✓ Answer ✓ Answer ✓ S/R (5) OR ✓ Statement ✓ Reason ✓ Answer ✓ S/R ✓ Answer (5)
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7.2	$x = 25^\circ$ $y = 50^\circ$ $\hat{C} = 90^\circ$ $z + y + 90^\circ = 180^\circ$ $z = 180^\circ - 90^\circ - 50^\circ$ $= 40^\circ$	\angle s in same seg. \angle s at centre = $2x \angle$ s at circumference radius \perp tangent Int \angle of Δ	✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ Answer (5)
7.3.1	$Q_1 = S_1 = 23^\circ$ $S_2 = Q_1 = 23^\circ$	Tan chord theorem \angle s opp equal sides	✓ Statement ✓ Reason ✓ Statement ✓ Reason (4)
7.3.2	$Q_1 + S_2 + R = 180^\circ$ $R = 180^\circ - 23^\circ - 23^\circ$ $= 134^\circ$	Int \angle of Δ	✓ S/R ✓ Answer (2)
7.3.3	$P + R = 180^\circ$ $P = 46^\circ$	opp \angle s of cyclic quad	✓ S/R ✓ Answer (2)
7.3.4	$O_1 = 92^\circ$	\angle s at centre = $2x \angle$ s at circumference	✓ Statement ✓ Reason (2) [20]



QUESTION 8

8.1	$AC^2 = AB^2 + BC^2$ $10^2 = AB^2 + 8^2$ $AB^2 = 36$ $AB = 6$ $DB = 3$	Pythagoras Midpt Theorem	✓ Substitution ✓ Answer ✓ Statement ✓ Reason (4)
8.2.1	Parallelogram		✓ Answer (1)
8.2.2 (a)	$\hat{A}FD = 41^\circ$ $\hat{B}ED = 41^\circ$ $\hat{E}DF = 41^\circ$	Corresponding Angles DF//EC Corresponding Angles DE//CF Opposite angles of parm.	✓ Statement ✓ Reason ✓ Statement ✓ Reason ✓ Statement ✓ Reason (6)
8.2.2 (b)	$\frac{EC}{BE} = \frac{AD}{BD}$ $\frac{EC}{6} = \frac{4}{5}$ $EC = 4.8cm$	line one side of Δ	✓ Statement ✓ Reason ✓ Substitution ✓ Answer (4)
8.2.2 (c)	$\frac{FC}{AF} = \frac{BD}{AD}$ $\frac{FC}{6} = \frac{5}{4}$ $FC = 7.5cm$	line one side of Δ	✓ S/R ✓ Substitution ✓ Answer (3) [18]

QUESTION 9

9.1	$Area = l \times b$ $= 15 \times 45$ $= 675$ $675 \times 2 = 1350m^2$	✓ Formula ✓ Answer ✓ Answer x 2 (3)
9.2	$Area_1 = l \times b$ $= 45 \times 8$ $= 360$ $Area_2 = l \times b$ $= 24 \times 8$ $= 192$ $a^2 = b^2 + c^2$ $15^2 = 12^2 + c^2$ $c^2 = 81$ $c = 9$ $Area_3 = \frac{1}{2} b \times h$ $= \frac{1}{2} (24)(9)$ $= 108$ $SA = 2(108 + 192 + 360)$ $= 1320m^2$	✓ Area of rectangle front ✓ Area of rectangle side ✓ Height of triangle ✓ Area of triangle ✓ Surface Area (5)
9.3	$24 \times 45 = 1080m^2$	✓ Answer (1)
9.4	$Volume = \frac{1}{2} b \times h \times H$ $= 108 \times 45$ $= 4860m^3$	✓ Formula ✓ Substitution ✓ Answer (3) [12]



QUESTION 10

10.1.1	$420 \div 60 = 7 \text{ Rev/s}$ $V = \pi DN$ $= \pi(0.8)(7)$ $= 17.59 \text{ m/s}$ $17.59 \times 3.6 = 63.33 \text{ km/h}$	✓ Conversion to rev/s ✓ Formula ✓ Substitution ✓ Answer ✓ Conversion km/h (5)
10.1.2	$1680 \div 60 = 28 \text{ rev/s}$ $\omega = 2\pi N$ $= 2\pi(28)$ $= 175.93 \text{ rad/s}$ $V = \omega r$ $17.59 = (175.93)r$ $r = 0.1 \text{ m}$	✓ Conversion to rev/s ✓ Formula ✓ Answer ✓ Formula ✓ Answer (5)
10.2	$145 \times \frac{\pi}{180} = 2.53 \text{ rad}$ $\text{Area} = \frac{1}{2} \theta r^2$ $= \frac{1}{2} (2.53)(15)^2$ $= 284.63 \text{ m}^2$ OR 284.71 m^2 if unrounded value for angle in radians was used.	✓✓ Convert to radials ✓ Formula ✓ Substitution ✓ Answer (5) [15]



QUESTION 11

11	$300 \div 6 = 50$ $Area = a \left(\frac{\sigma_1 + \sigma_n}{2} + \sigma_2 + \sigma_3 + \dots \sigma_{n-1} \right)$ $= 50 \left(\frac{140 + 0}{2} + 160 + 200 + 190 + 180 + 130 \right)$ $= 46500 \text{ unit}^2$	✓ Widths of intervals ✓ Formula Substitution: ✓ $\sigma_1 = 140$ ✓ $\sigma_n = 0$ ✓ $160 + 200 + 190 + 180 + 130$ ✓ Answer [6]
	GRAND TOTAL	150

