Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sport Ontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko
NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

SEPTEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages and an information sheet consisting of 2 pages and 2 and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

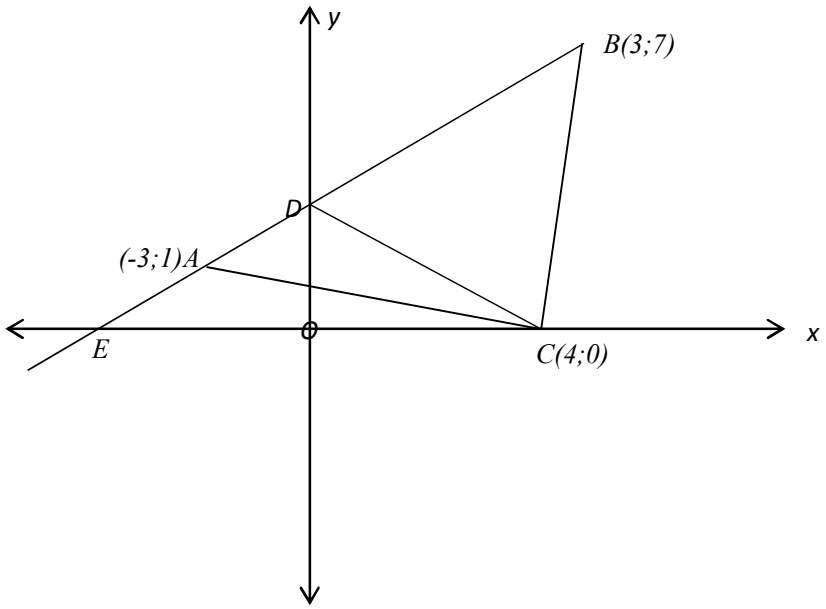
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessary be awarded full marks.
5. You may use an approved scientific calculator (non ï programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places unless stated otherwise.
7. Diagrams are NOT necessary drawn to scale.
8. Diagram sheets for answering question 2.2 and 5.1 are provided.
9. An information sheet , with formulae, is included at the end of the question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write legibly and neatly.

QUESTION 1

In the diagram below A(-3; 1), B(3; 7) and C(4; 0) are vertices of Δ ABC.

Use the diagram to answer the questions that follow.



1.1 Determine the equation of AB. (4)

1.2 Proof that OD is 4 units. (2)

1.3 Use analytical methods to proof that DC is perpendicular to AB. (4)

1.4 Determine the angle of inclination that DC makes with the x axis. (3)

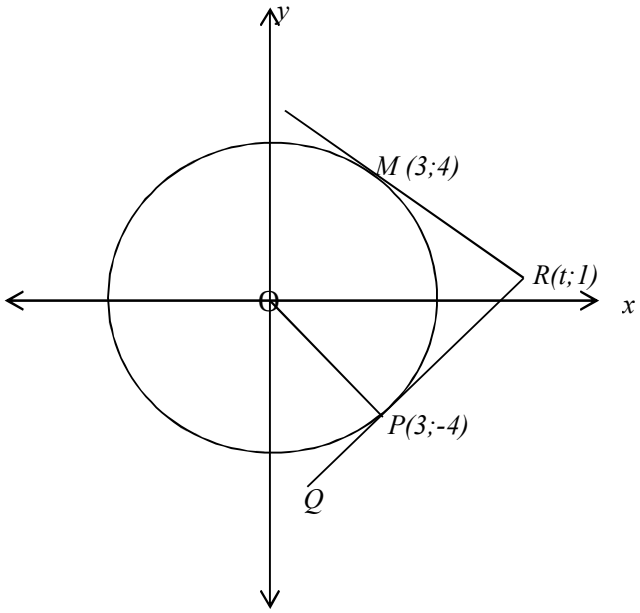
1.5 Determine the coordinates of E. (2)

1.6 If the length of BC is $5\sqrt{2}$ determine which type of triangle ΔABC is. (3)

[18]

QUESTION 2

2.1 In the diagram below O is the centre of a circle and QR and MR are both tangents to the circle. P (3; -4) and R(t ; 1) are points on line QR.



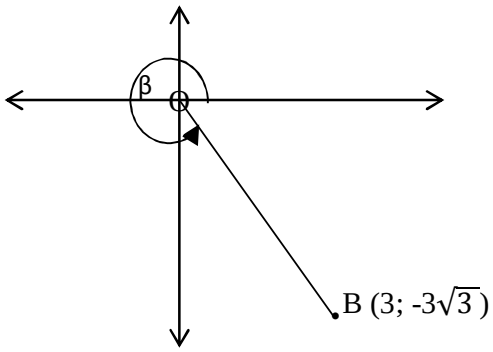
Determine

2.1.1 The equation of the circle in the form $x^2 + y^2 = r^2$ (2)

2.1.2 The equation of RQ the tangent to the circle at P. (4)

2.2 Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{49} = 1$ (On the grid provided)
Clearly show ALL the intercepts with the axes. (3)
[9]

QUESTION 3



3.1 Use the diagram above and determine without using a calculator the numerical value of:

3.1.1 OB (2)

3.1.2 $sec^2\beta$ (3)

3.1.3 $3cot^2\beta - sin^2\beta$ (3)

3.2 Determine the numerical value of $cos\frac{2\pi}{3}$. (2)

3.3 Calculate the numerical value of $cosec\,x$ if $15cos\,x = -12$, if $x \in [0°;180°]$ (4)

3.4 If $tan\theta = -\frac{2}{5}$ and $sin\theta > 0$ for [illegible] $[0°; 180°]$

3.4.1 In which quadrant will [illegible] lie? (1)

3.4.2 Hence determine the size of θ . (4)

[19]

QUESTION 4

4.1 Use reduction formulae to simplify the following to a single trigonometric ratio:

$$\frac{tan^2(180° - x)cos^2(180° + x)cosec(360° - x)}{sin(180° + x)}$$ (8)

4.2 Complete the following identity [illegible] $sec^2 2x - tan^2 2x = \ldots$ (1)

4.3 Simplify the following to one trigonomic ratio :

$$sin^2\theta + \frac{1}{sec^2\theta} + \frac{tan\theta}{cot\theta}$$ (7)

[16]

QUESTION 5

Given $f(x)=\frac{1}{2}sinx$ and $g(x)=-\cos x$ $x \in [0°;360°]$

5.1 Draw a neat sketch of both *f* and *g* on the same set of axis on the grid provided. Clearly showing the turning points, end points and intercepts (6)

5.2 Use the graphs to answer the questions below:

5.2.1 What is amplitude of *g* ? (1)

5.2.2 What is the period of *f* ? (1)

5.2.3 Use the graph to determine for which values of x $f(x) = g(x)$ (2)

5.2.4 Determine the values (in interval notation) of x for which $f(x) \geq g(x)$ for $x \in [0°;360°]$ (2)

[12]

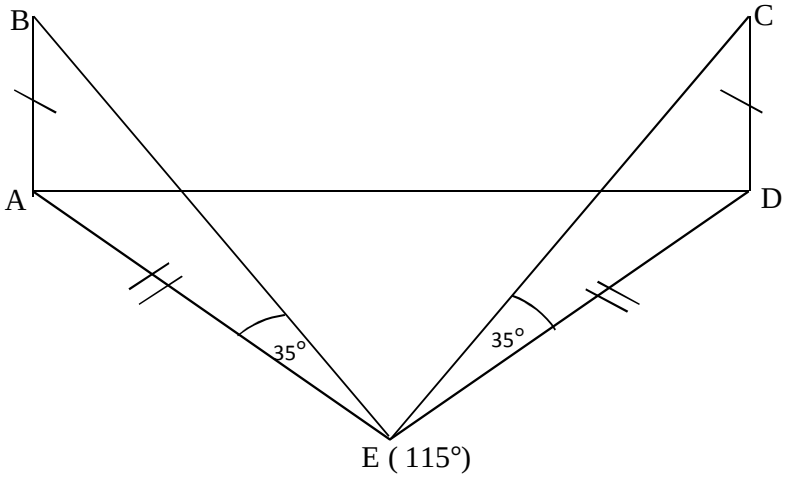
QUESTION 6

Two buildings, AB and CD are at the same height and stand on the same horizontal plane as shown on the figure below.
E is on the same horizontal plane as A and D. AE and DE are both 15m long.
The angle of elevation of both B and C measured from E are $35°$. $A\hat{E}D = 115°$.

Answer the following questions rounded off to two decimal places correctly.

6.1 Determine the height of the two buildings. (3)

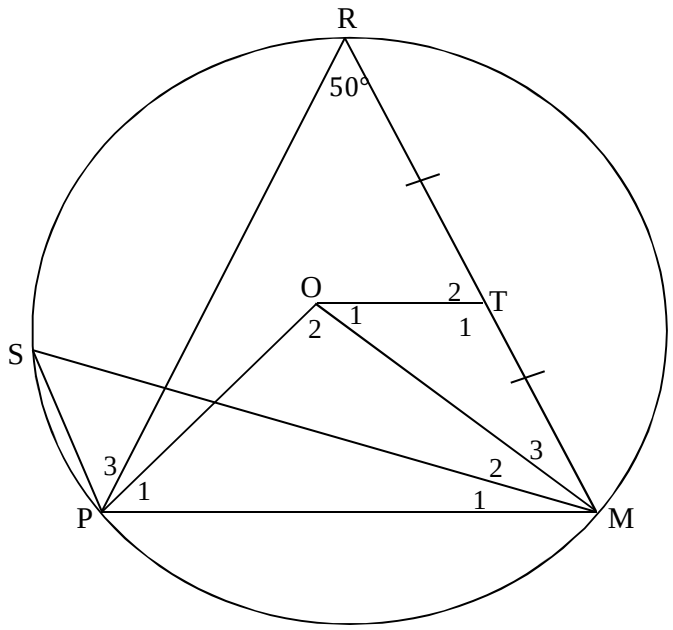
6.2 Determine the distance between the two buildings. (3)



[6]

QUESTION 7

7.1 The diagram below shows a circle with centre O. $P\hat{R}M = 50°$ and RT =TM.



Determine with reasons the sizes of the following angles:

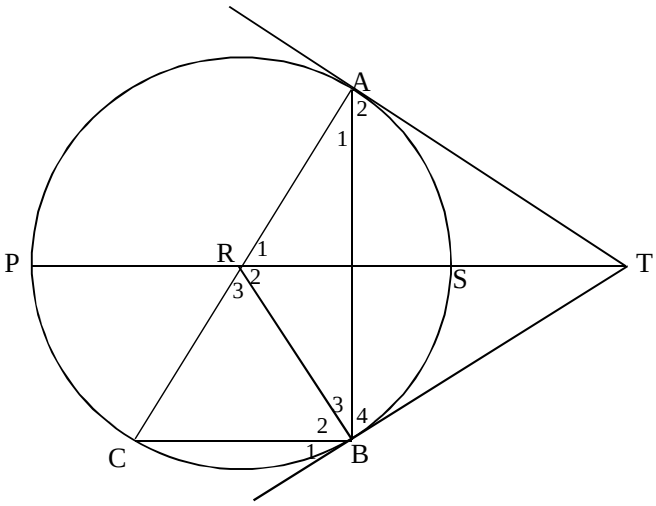
7.1.1 $\hat{O}_2$ (2)

7.1.2 $\hat{P}_1$ (3)

7.1.3 $\hat{S}$ (2)

7.2 TA and TB are both tangents. C is a point on the circumference so that BC is parallel to secant TSP. BR intersects AC at Rand R is the centre of the circle. RB and AB are connected. $\hat{A}_2 = 60°$ and PT || CD.

Use the sketch and information given to answer the questions below.



7.2.1 Give a reason why $RBT = 90°$ (1)

7.2.2 If $A_2 = 60°$, name with reasons FOUR other angles also equal to $60°$. (4)

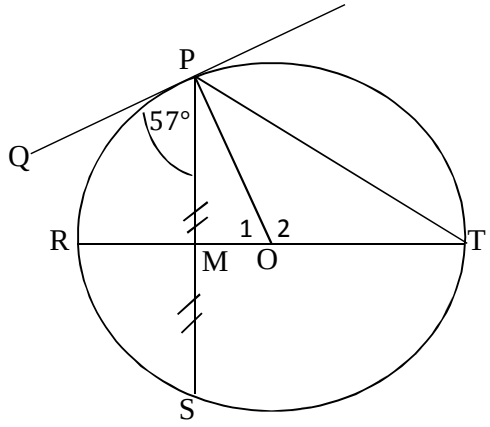
7.2.3 Determine with reason the size of A_1 (2)

7.2.4 Determine the size of RTA. (2)

7.2.5 What type of quadrilateral is *RATB*? Give a reason for your answer. (2)

7.3 In the sketch below $\widehat{P}_4 = 57°$. QP is a tangent to circle at P. RT bisects PS at M. O is the centre of the circle.

Use the information to answer the questions below.



7.3.1 Complete the following statement:

If the diameter bisects a chord the diameter is _____ to the chord. (1)

7.3.2 Determine the following, give reasons for your answers:

a) $S\hat{P}O$ (2)

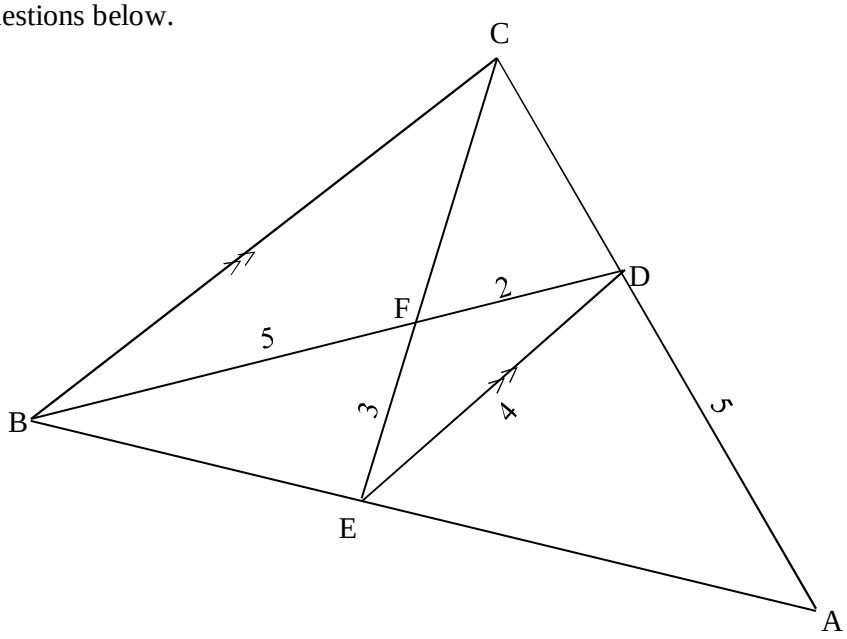
b) $O\hat{T}_1$ (2)

[23]

QUESTION 8

8.1 Give two reasons for triangles to be similar. (2)

8.2 In Δ□□□, □□ || □□. Use the sketch and the given information to answer the questions below.



8.2.1 Name with a reason a triangle similar to Δ DFE. (1)

8.2.2 It is given that AD = 5 units, DE = 4 units, DF = 2 units, BF = 5 units and EF = 3 units. Calculate the lenghts of :

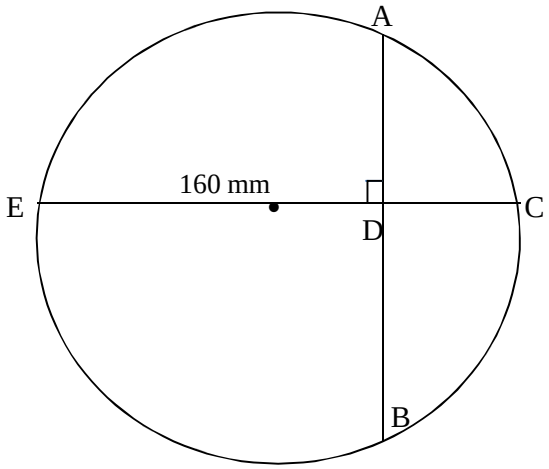
a) BC (3)

b) FC (2)

[8]

QUESTION 9

9.1 A circle with a diameter of 240 mm is divided into two segments by chord AB, as shown in the diagram below.
DE, the height of the major segment is 160 mm.



Calculate:

9.1.1 the radius of the circle (1)

9.1.2 the value of *h,* the height of the minor segment (1)

9.1.3 the length of chord AB. (3)

9.2 A point on the circumference of a driver wheel driving a belt moves at a speed of 3 m/s. The diameter of the driver wheel is 0,6 m.

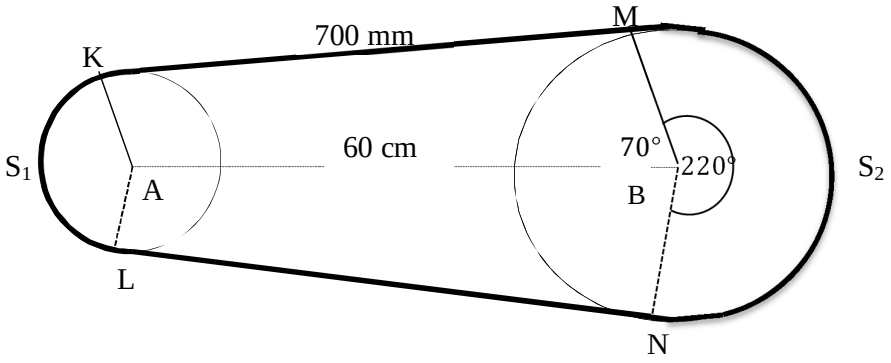
Calculate:

9.2.1 the distance covered by this point in 20 s (2)

9.2.2 the number of revolutions per second (3)

9.2.3 the time it takes the wheel to complete 12 revolutions. (2)

9.3 An engineer designs a pulley system with two pulleys, the smaller pulley has a radius of 100 mm and the larger one a diameter of 400 mm. The centres, A and B are 60 cm apart. KM is 700 mm.



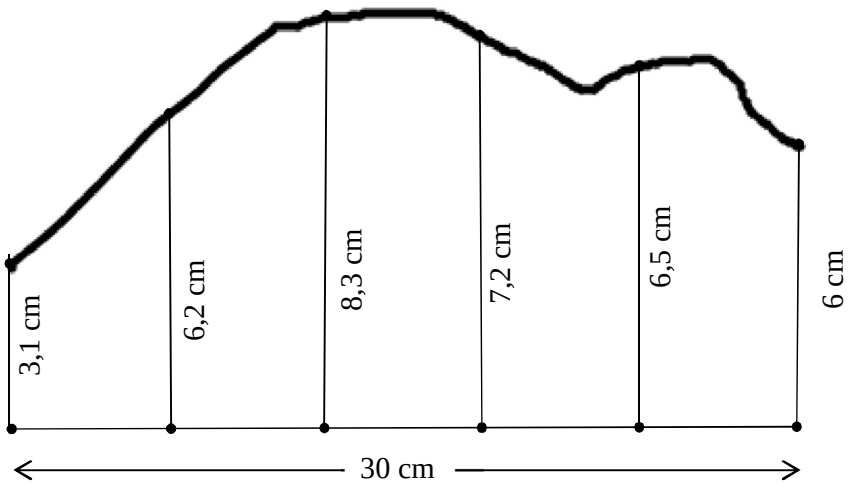
9.3.1 Determine the magnitude of $\vec{AB}$ if $\vec{AB} \parallel \vec{LN}$ (2)

9.3.2 If the length of S_1 is 38,4 cm, determine the length of the driver belt in cm, rounded to one decimal places. (8)

[22]

QUESTION 10

10.1 The irregular shape below has one straight side of 30 cm divided into 5 equal parts. The ordinates dividing the parts are: 3,1 cm; 6,2 cm; 8,3 cm; 7,2 cm; 6, 5 cm and 6 cm.



10.1.1 Determine the value of *a* the length of the equal parts. (1)

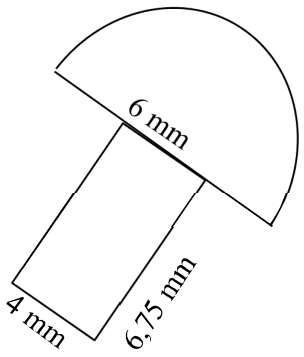
10.1.2 Calculate the approximate area of the irregular shape using the mid-ordinate rule. (4)

10.2 A solid piece of metal in the form of a cube, with a length of 70 mm, is melted down and then rivets are recast. The rivets consist of a cylinder and a head in the form of a hemisphere as indicated in the sketch.

Determine :

10.2.1 the volume of metal required to cast one rivet, rounded to 2 decimal places. (8)

10.2.2 the number of rivets that can be casted from one piece of metal. (4)



The following formulas may be used

$Volume\ of\ a\ cylinder = \pi r^2 h$

$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$

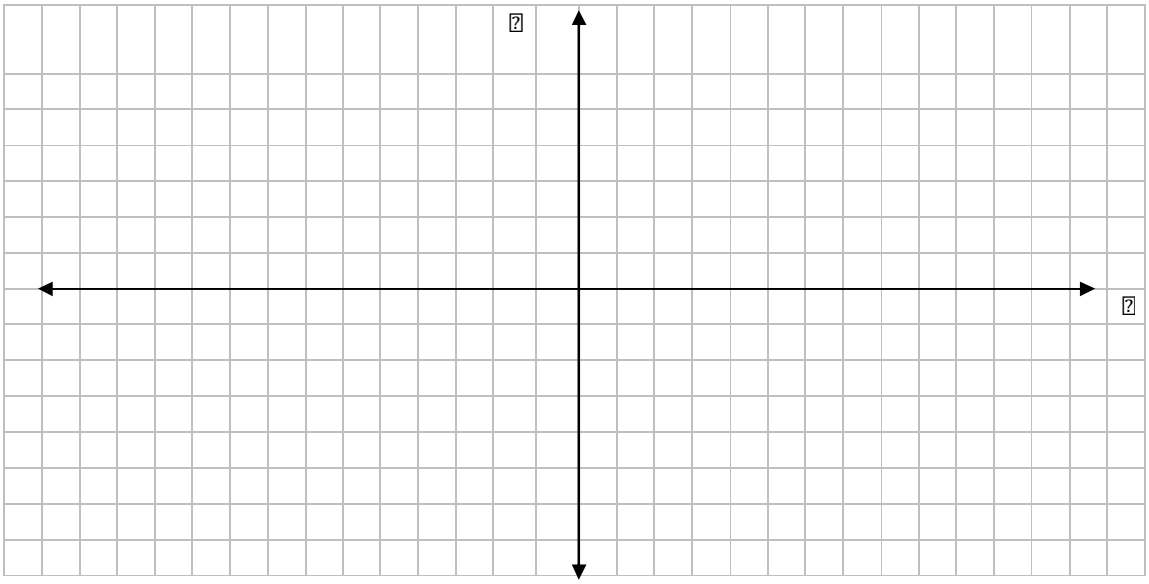
[17]

TOTAL: 150

ANSWER SHEET

Name:.....

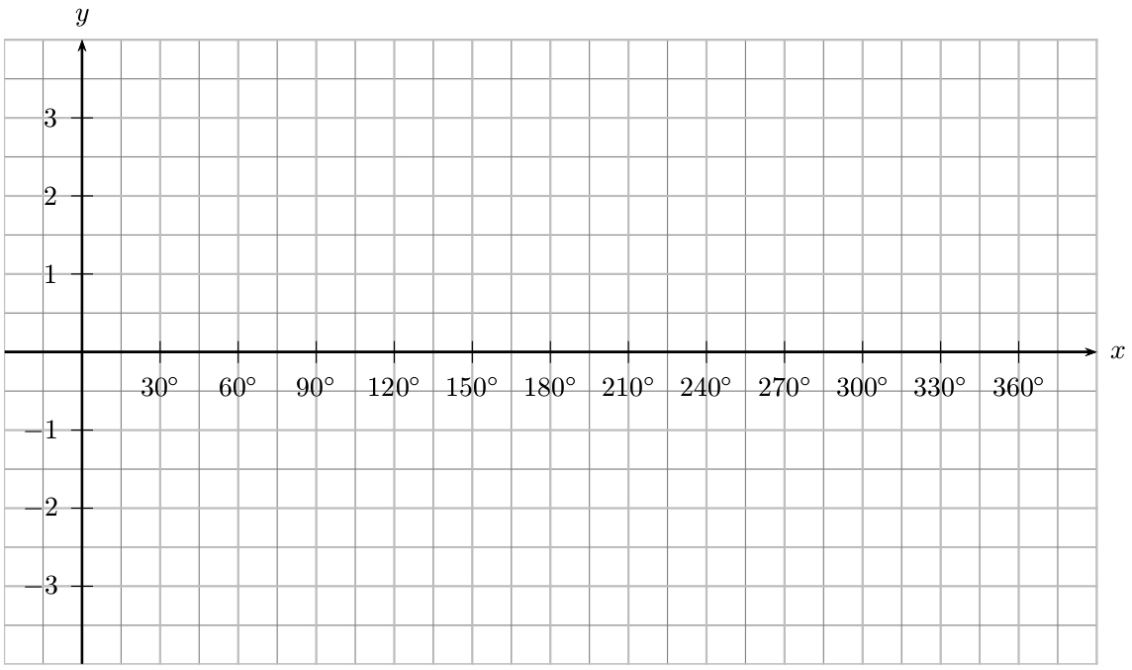
QUESTION 2.2



ANSWER SHEET

Name:.....

QUESTION 5.1



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = \log_a b$, $a > 0$, $a \neq 1$ and $b > 0$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$i_{eff} = \left(1 + \frac{i^m}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ of\ \Delta\text{ABC} = \tfrac{1}{2}ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \cos ec^2\theta$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln(x) + C \text{ , } \quad x > 0 \qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$\pi rad = 180°$

$\text{Angular velocity} = \omega = 2\pi n = 360^0 n$ where n = rotation frequency

$\text{Circumferencial velocity} = v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

$\text{Area of a sector} = \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$